

**Problem 1**

(4 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 60 minutes, put away your notes and calculator, and solve the exam in one sitting.

On the real exam, you will earn 4 points for correctly filling out the scantron. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded.)

Answer = \_\_\_\_\_

**Problem 2**

(8 points)

The extreme values of  $f(x,y,z) = 3x + 2y + 6z$  with constraint  $x^2 + y^2 + z^2 = 4$  are.

To receive the full 5 points, you must show all your work on this problem.

- A. The maximum of  $f$  is 7 and the minimum of  $f$  is -7
- B. The maximum of  $f$  is 28 and the minimum of  $f$  is -28
- C. The maximum of  $f$  is 14 and the minimum of  $f$  is -7
- D. The maximum of  $f$  is 7 and the minimum of  $f$  is -14
- E. The maximum of  $f$  is 14 and the minimum of  $f$  is -14

**Problem 3**

(8 points)

Compute the double integral  $\iint_R \cos(x + y) dA$ , where  $R$  is the rectangle  $[0, \pi] \times [0, \pi]$ .

- A. -2
- B. -4
- C. 2
- D. 0
- E. 4

**Problem 4**

(8 points)

Reverse the order of integration and evaluate the double integral:  $\int_0^1 \int_{x^2}^1 6\sqrt{y} \cos(y^2) dy dx$

- A.  $\sin(1)$
- B.  $3 \cos(1)$
- C.  $2 \cos(1) - 2$
- D.  $3 \sin(1)$
- E.  $2 \sin(1)$

**Problem 5**

(8 points)

Let  $D$  be the region in the **first quadrant** between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ . Evaluate the

$$\text{integral} \iint_D \frac{x^2 y}{(x^2 + y^2)^{3/2}} dA.$$

- A. 14/3
- B. 1/2
- C. 10/3
- D. 5/6
- E. 3/2

**Problem 6**

(8 points)

Compute the triple integral

$$\iiint_E 3y dV,$$

where  $E$  is a region under the plane  $x + y + z = 2$  in the first octant.

- A. 1
- B. 3
- C. 6
- D. 4
- E. 2

**Problem 7**

(8 points)

The integral

$$\int_0^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{8-x^2-y^2}} xy^2 z dz dy dx$$

when converted to cylindrical coordinates becomes

- A.  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \int_{\sqrt{3r}}^{\sqrt{8-r^2}} r^4 z \cos \theta \sin^2 \theta dz dr d\theta$
- B.  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \int_{\sqrt{3r}}^{\sqrt{8-r^2}} r^3 z \cos \theta \sin^2 \theta dz dr d\theta$
- C.  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \int_{\sqrt{3r}}^{\sqrt{8-r^2}} r^4 z \cos \theta \sin^2 \theta dz dr d\theta$
- D.  $\int_0^{\pi} \int_0^{\sqrt{2}} \int_{\sqrt{3r}}^{\sqrt{8-r^2}} r^4 z \cos \theta \sin^2 \theta dz dr d\theta$
- E.  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \int_{\sqrt{3r}}^{\sqrt{8-r^2}} r^4 z \cos \theta \sin^2 \theta dz dr d\theta$

**Problem 8**

(8 points)

Compute  $\iiint_E z dV$ , where  $E$  is bounded by the sphere  $x^2 + y^2 + z^2 = 1$  and the coordinate planes in the first octant.

- A.  $\frac{3\pi}{8}$
- B.  $\frac{\pi}{16}$
- C.  $\frac{8}{\pi}$
- D.  $\frac{\pi}{12}$
- E.  $\frac{\pi}{6}$

**Problem 9**

(8 points)

A lamina with density  $\rho(x,y) = xy$  occupies the region of the plane bounded by  $y = x^2$ ,  $y = 1$  and  $x = 0$ . The mass of lamina is equal to  $\frac{1}{6}$ . Find the y-coordinate of its center of mass.

- A.  $2/3$
- B.  $3/4$
- C.  $5/6$
- D.  $12/21$
- E.  $7/8$

**Problem 10**

(8 points)

Let  $f(x,y,z) = x^2 + y^3 + z^4$  and  $g(x,y,z) = 3x + 4y + \frac{z^2}{2}$ . If  $\vec{\nabla} f(2,1,-1)$  is perpendicular to  $\vec{\nabla} g(a,b,c)$ , then

- A.  $c = 10$
- B.  $c = 6$
- C.  $c = 2$
- D.  $c = 8$
- E.  $c = 4$

**Problem 11**

(8 points)

Compute the line integral  $\int_C (4x^3 + y^3) ds$ , where  $C$  is the line segment from  $(0,0)$  to  $(1,2)$ .

- A. 0
- B.  $3\sqrt{5}$
- C.  $\sqrt{5}\pi$
- D. -5
- E.  $\frac{5\sqrt{5}}{4}$

**Problem 12**

(8 points)

Evaluate the line integral  $\int_C xy dx - y^2 dy$ , where  $C$  is the line segment from  $(0,0)$  to  $(2,6)$ .

- A. 42
- B. 36
- C. -64
- D. -44
- E. -36

**Problem 13**

(8 points)

Compute the line integral  $\int_C F \cdot dr$ , where  $F = \langle yz, xz, xy \rangle$  and the curve  $C$  is parametrized by  $r(t) = \langle t^2, t, t^3 - 3t \rangle$ ,  $1 \leq t \leq 2$ .

- A. 18
- B. 10
- C. -16
- D. 0
- E.  $8\pi$

## Answer Key

PROBLEM	ANSWER
1.	<i>Name</i>
2.	E. The maximum of $f$ is 14 and the minimum of $f$ is -14
3.	B. -4
4.	D. $3 \sin(1)$
5.	B. $1/2$
6.	E. 2
7.	A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\sqrt{2}} \int_{\sqrt{3}r}^{\sqrt{8-r^2}} r^4 z \cos \theta \sin^2 \theta dz dr d\theta$
8.	B. $\frac{\pi}{16}$
9.	B. $3/4$
10.	B. $c = 6$
11.	B. $3\sqrt{5}$
12.	C. -64

**PROBLEM**

**ANSWER**

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13.

A. 18

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