Problem 1 (0 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 120 minutes, put away your notes and calculator, and solve the exam in one sitting.

The real Final Exam will have 20 questions with each question worth 5 points. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded)

Problem 2 (5 points)

A line l passes through the point $(-1,1,2)$ and is perpendicular to the plane $x - 2y + 2z = 8$. At what point does this line intersect with the yz-plane?

To receive the full 5 points, you must show all your work on this problem.

A. (0, 4, -1)
B. (0, 4 ,6)
C. (0, -1, 4)
D. (0, 1, 4)
E. (0, 3, 1)

Problem 3 (5 points)

Identify the surface $2x^2 + 3z^2 = 4x + 2y^2$ through completing the square.

A. Cone
B. Hyperboloid of two sheets
C. Ellipsoid
D. Parabolic hyperboloid
E. Hyperboloid of one sheet

Problem 4 (5 points)

Let (a,b,c) be the point of intersection of the space curve $\vec{r}(t) = \langle \sqrt{2}t, t^2+1, 1-4t \rangle$ with the surface $x^2+2y-z=0$. What is the value of a^2+2b ?

To receive the full 5 points, you must show all your work on this problem.

A. 3
B. 4
C. 5
D. 7
E. 6

Problem 5 (5 points)

Calculate the arc length of $r(t) = \langle 3\sin(2t), 4, 3\cos(2t) \rangle$ for $0 \le t \le \frac{\pi}{3}$.

A. 2π
B. π
C. $\frac{5\pi}{3}$
D. $-\frac{\pi}{3}$
E. 6π

Problem 6 (5 points)

The level curves of $f(x,y) = \sqrt{x^2+4y^2+4} - x$ are

A. sometimes lines and sometimes ellipses
B. parabolas
C. hyperbolas
D. ellipses
E. circles

Problem 7 (5 points)

If $f(x,y) = x\sin(xy^2)$, compute $f_{yx}(\pi,1)$.

To receive the full 5 points, you must show all your work on this problem.

A. -4π
B. -8π
C. $-\pi$
D. -6π
E. -2π

Problem 8 (5 points)

Find the directional derivative of $f(x,y) = xe^{y^2} + e^{x+y}$ at the point (0,0) in the direction of the vector $3\vec{i} - 4\vec{j}$.

To receive the full 5 points, you must show all your work on this problem.

A. 0
B. -6/5
C. 2/5
D. -2/5
E. 6/5

Problem 9 (5 points)

The shortest distance from (1,1,0) to the plane $x + y + z = 1$ is

A. $\sqrt{3}$
B. $\frac{\sqrt{3}}{4}$
C. 3
D. $\frac{\sqrt{3}}{2}$
E. $\frac{\sqrt{3}}{3}$

Problem 10 (5 points)

The extreme values of $f(x,y,z) = 3x + 2y + 6z$ with constraint $x^2 + y^2 + z^2 = 4$ are.

To receive the full 5 points, you must show all your work on this problem.

A. The maximum of f is 28 and the minimum of f is -28
B. The maximum of f is 7 and the minimum of f is -7
C. The maximum of f is 14 and the minimum of f is -14
D. The maximum of f is 14 and the minimum of f is -7
E. The maximum of f is 7 and the minimum of f is -14

Problem 11 (5 points)

Compute the double integral $\iint\limits_R \cos(x+y)\,dA$, where R is the rectangle $[0,\pi]\times[0,\pi]$.

A. -2
B. -4
C. 0
D. 2
E. 4

Problem 12 (5 points)

Which of the following integrals represents the volume of the solid in the first octant

that is bounded on the side by the surface $x^2+y^2=4$ and on the top by the surface $x^2+y^2+z=4$?

A. $\int_0^2\int_0^2\int_0^{4-x^2-y^2} dzdydx$
B. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} dzdydx$
C. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{x^2+y^2} dzdydx$
D. $\int_0^4\int_0^4\int_0^{4-x^2-y^2} dzdydx$
E. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{4-z} dzdydx$

Problem 13 (5 points)

Compute $\iiint_E zdV$, where E is bounded by the sphere $x^2+y^2+z^2=1$ and the coordinate planes in the first octant.

A. $\frac{3\pi}{8}$
B. $\frac{\pi}{12}$
C. $\frac{\pi}{8}$
D. $\frac{\pi}{6}$
E. $\frac{\pi}{16}$

Problem 14 (5 points)

Let C be the half circle $x^2 + y^2 = 4$ with $x \geq 0$ then

$$\int_C x ds =$$

A. 0
B. 8
C. 4
D. 2
E. 1

Problem 15 (5 points)

Compute the line integral $\int_C F \cdot dr$, where $F = \langle yz, xz, xy \rangle$ is conservative and the curve C is parametrized by $r\,(t) = \left\langle t^2, t, t^3 - 3t \right\rangle$, $1 \leq t \leq 2$.

A. 18
B. 8π
C. 0
D. 10
E. -16

Problem 16 (5 points)

Compute

$$\int_C \left(e^{2x} + y^2\right) dx + \left(14xy + y^2\right) dy,$$

where C is the boundary of the region bounded by the y-axis and the curve $x = y - y^2$ oriented counterclockwise.

A. 2
B. 1
C. 4
D. 12
E. 24

Problem 17 (5 points)

Find grad(div(F)) ·curl(F) for $F(x,y,z) = xy\vec{i} + yz\vec{j} + xz\vec{k}$ at (1,−1,2)

A. 0

B. 3

C. -2

D. 1

E. -4

Problem 18 (5 points)

Find the area of the part of the plane $3x + 2y + z = 6$ that is in the first octant.

A. $3\sqrt{20}$

B. $3\sqrt{22}$

C. $3\sqrt{14}$

D. $3\sqrt{10}$

E. $3\sqrt{6}$

Problem 19 (5 points)

The flux of the vector field $\vec{F}(x,y,z) = x\vec{i} + (x+y)\vec{j} + z\vec{k}$ across the surface of the plane $x + y + z = 1$ in the first octant, oriented upward, is equal to:

A. 3/4

B. 3/2

C. 4/3

D. 1/2

E. 2/3

Problem 20 (5 points)

Use Stokes' Theorem to evaluate the integral $\int_C y\,dx + z\,dy + x\,dz$, where C is the intersection of the surfaces $x^2 + y^2 = 1$ and $x + y + z = 5$. C is oriented counterclockwise when viewed from above.

A. -6π

B. -3π

C. -9π

D. -8π

E. $-\pi$

Problem 21 (5 points)

Evaluate the flux integral $\iint_S \vec{F} \cdot dS$ where $F(x,y,z) = \langle 3xy^2, x\cos(z), z^3 \rangle$ and S is the complete boundary surface of the solid region bounded by the cylinder $y^2 + z^2 = 2$ and the planes $x = 1$ and $x = 3$. S is oriented by the outward normal.

A. 12π

B. 9π

C. 14π

D. 24π

E. 18π

Answer Key

PROBLEM	POINTS	ANSWER
1.	0	$Name$
2.	5	**C.** (0, -1, 4)
3.	5	**E.** Hyperboloid of one sheet
4.	5	**A.** 3
5.	5	**A.** 2π
6.	5	**B.** parabolas
7.	5	**A.** -4π
8.	5	**C.** 2/5
9.	5	**E.** $\frac{\sqrt{3}}{3}$
10.	5	**C.** The maximum of f is 14 and the minimum of f is -14
11.	5	**B.** -4
12.	5	**B.** $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} dzdydx$

PROBLEM	POINTS	ANSWER
13.	5	**E.** $\frac{\pi}{16}$
14.	5	**B.** 8
15.	5	**A.** 18
16.	5	**B.** 1
17.	5	**C.** -2
18.	5	**C.** $3\sqrt{14}$
19.	5	**E.** 2/3
20.	5	**B.** -3π
21.	5	**A.** 12π