Problem 1 (0 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 120 minutes, put away your notes and calculator, and solve the exam in one sitting.

The real Final Exam will have 20 questions with each question worth 5 points. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded)

Problem 2 (5 points)

Find an equation of the plane that contains the point (1, 2, −3) and the line with symmetric equations $x-2=y-1=\dfrac{z+2}{2}$.

To receive the full 5 points, you must show all your work on this problem.

A. $4x-2y-3z=9$

B. $3x+y-2z=11$

C. $2x-y+z=-3$

D. $x+y-2z=9$

E. $5x+y+z=4$

Problem 3 (5 points)

Identify the surface defined by the equation $x^2+y^2+2z-z^2=0$.

A. Hyperboloid of two sheets
B. Paraboloid
C. Hyperboloid of one sheet
D. Ellipsoid
E. Ellipse

Problem 4 (5 points)

A particle has position $\vec{r}(t)$ with acceleration $\vec{a}(t) = t\vec{i} + 3t^2\vec{k}$ and the initial conditions $\vec{v}(0) = \vec{i} + \vec{j} + \vec{k}$ and $\vec{r}(0) = \vec{0}$. Then $\vec{r}(1) =$

To receive the full 5 points, you must show all your work on this problem.

A. $\frac{1}{6}\vec{i} + \frac{1}{4}\vec{k}$
B. $\vec{i} + \vec{j} + \vec{k}$
C. $5\vec{i} + 7\vec{j} + \vec{k}$
D. $\vec{i} + \frac{5}{4}\vec{k}$
E. $\frac{7}{6}\vec{i} + \vec{j} + \frac{5}{4}\vec{k}$

Problem 5 (5 points)

Find the length of the curve $r(t) = \langle \sin t - t\cos t, \cos t + t\sin t\rangle, 0 \le t \le 2\pi$.

A. 2π
B. $2\pi^2$
C. $\frac{\pi^2}{2}$
D. $4\pi^2$
E. π^2

Problem 6 (5 points)

The level curves of $f(x,y) = \sqrt{x^2 + y^2 + 1 + x}$ are

A. hyperbolas
B. parabolas
C. ellipses
D. sometimes lines sometimes ellipses
E. circles

Problem 7 (5 points)

Suppose that z is defined as a function of x and y by the equation

$\cos(xyz) = x + 3y + 2z.$

Use implicit differentiation to find the value of $\dfrac{\partial z}{\partial y}$ $(0{,}1)$.

A. -2/3
B. -1/2
C. -3/5
D. -3/2
E. 1/3

Problem 8 (5 points)

Find the maximum rate of change of $f(x{,}y) = \sqrt{7 - x^2 - y^2}$ at the point (-2, 1).

To receive the full 5 points, you must show all your work on this problem.

A. $\dfrac{3}{\sqrt{2}}$
B. $\dfrac{5}{\sqrt{2}}$
C. $\sqrt{8}$
D. $\dfrac{\sqrt{10}}{2}$
E. 1/4

Problem 9 (5 points)

The function $f(x{,}y) = 6x^2 + 3y^2 - 16$ attains its local minimum at:

A. (6,0)
B. (6,-3)
C. (3,0)
D. (6,3)
E. (0,0)

Problem 10 (5 points)

Find the minimum value of $f(x,y) = 2x + 3y + 2$ given that $2x^2 + 5xy + 4y^2 = 28$

To receive the full 5 points, you must show all your work on this problem.

A. -6
B. -1
C. -2
D. -8
E. -3

Problem 11 (5 points)

Reverse the order of integration and evaluate the double integral: $\int_0^1 \int_{x^2}^1 6\sqrt{y}\cos\left(y^2\right)\, dy\, dx$

A. $3\cos(1)$

B. $2\sin(1)$

C. $2\cos(1) - 2$

D. $3\sin(1)$

E. $\sin(1)$

Problem 12 (5 points)

Evaluate the triple integral $\iint\int_V 2z dV$, where V is bounded by $z = 2 - x^2 - y^2$ and $z = 1$.

A. π

B. $\frac{2\pi}{3}$

C. $1 + \frac{2\pi}{3}$

D. $\frac{4\pi}{3}$

E. $1 + \frac{4\pi}{3}$

Problem 13 (5 points)

Convert the integral to spherical coordinates and compute it:

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} 3dzdy\,dx$$

A. $2\left(\sqrt{2}-1\right)\pi$

B. $10\left(\sqrt{2}-1\right)\pi$

C. $12\left(\sqrt{2}-1\right)\pi$

D. $8\left(\sqrt{2}-1\right)\pi$

E. $16\left(\sqrt{2}-1\right)\pi$

Problem 14 (5 points)

Let C be the curve **r**(t)$=<\cos t, \sin t, t>$, $t \in [0, \frac{\pi}{2}]$ and $f(x,y,z) = xy$ then

$$\int_C f(x,y,z)ds =$$

A. 1/2

B. 0

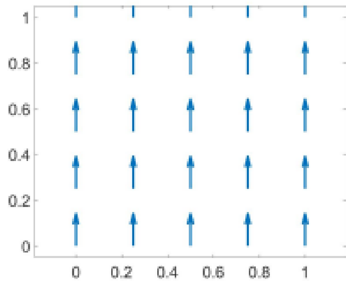
C. $\sqrt{2}$

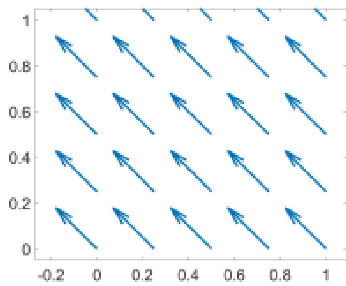
D. 1

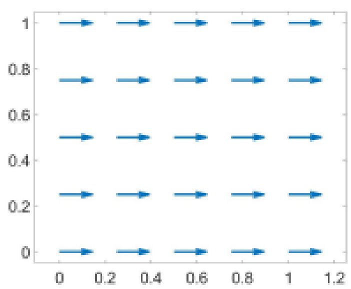
E. $\frac{1}{\sqrt{2}}$

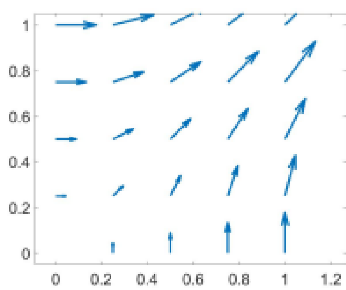
Problem 15 (5 points)

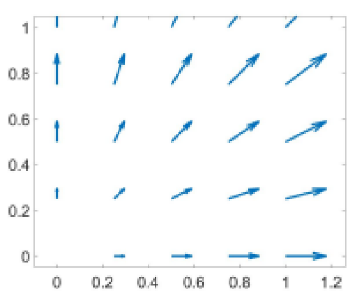
A particle is traveling on the path $y = x$ from (0,0) to (1,1). For which of the following force vector fields is the work done equal to 0?

A. 

B. 

C. 

D. 

E. 

Problem 16 (5 points)

According to Green's Theorem, which of the following line integrals is **NOT** equal to the area of the region enclosed by a simple curve C?

A. $\frac{1}{2}\int_C (-y\,dx + x\,dy)$

B. $\frac{1}{5}\int_C (4y\,dx - x\,dy)$

C. $\frac{1}{3}\int_C (y\,dx + 4x\,dy)$

D. $\int_C -y\,dx$

E. $\int_C xdy$

Problem 17 (5 points)

Find grad(div(F)) ·curl(F) for $F(x,y,z) = xy\vec{i} + yz\vec{j} + xz\vec{k}$ at (1,−1,2)

A. -4

B. 0

C. 3

D. 1

E. -2

Problem 18 (5 points)

Find the surface area of the parametric surface

$$r(u,v) = \langle 2u + 3v, 3u + v, 2\rangle, \quad 0 \le u \le 2, \quad 0 \le v \le 1.$$

A. 4

B. 14

C. 12

D. $3\sqrt{2}$

E. $4\sqrt{2}$

Problem 19 (5 points)

Consider $F = \frac{r}{|r|^3}$ where $r = \langle x,y,z \rangle$ and $|r| = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}$. Which one of the following is true.

(i) $\int_C F \cdot dr$ is independent of path.

(ii) $\int\int_S F \cdot ndS = 0$ for any closed surface S that encloses the origin.

(iii) $div\,(F) = 0$

A. only (ii) and (iii)
B. All of the above
C. None of the above
D. only (i) and (iii)
E. only (i) and (ii)

Problem 20 (5 points)

Let $F = (y + zcos\,(x))i + (-x + z\sin(y))j + \left(xye^z\right)k$, compute

$$\iint_S (\nabla \times F) \cdot ndS,$$

Where S is the part of the graph of $z = f\,(x,y) = e^x\,\left(x^2 + y^2 - 36\right)$ below the xy-plane

with downward pointing normal.

A. -36π

B. 72π

C. 0

D. 36π

E. -72π

Problem 21 (Corrected) (5 points)

Compute

$$\iint_S F \cdot n dS$$

the net outward flux of the vector field $F = \langle x + y, y - z, xy + z \rangle$ across the surface S, which is the boundary of the solid bounded by $z = 0, y = 0, y + z = 2,$ and (correction) $z = 1 - x^2$.

A. −16/5

B. −32/15

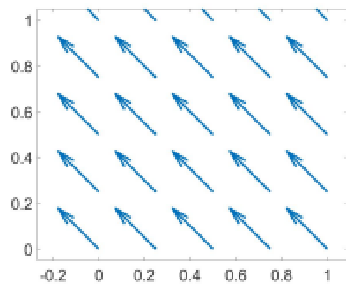
C. 32/5

D. −32/5

E. 32/15

Answer Key

PROBLEM	POINTS	ANSWER
1.	0	$Name$
2.	5	**B.** $3x + y - 2z = 11$
3.	5	**A.** Hyperboloid of two sheets
4.	5	**E.** $\frac{7}{6}\vec{i} + \vec{j} + \frac{5}{4}\vec{k}$
5.	5	**B.** $2\pi^2$
6.	5	**B.** parabolas
7.	5	**D.** -3/2
8.	5	**D.** $\frac{\sqrt{10}}{2}$
9.	5	**E.** (0,0)
10.	5	**A.** -6
11.	5	**D.** $3\sin(1)$
12.	5	**D.** $\frac{4\pi}{3}$

PROBLEM	POINTS	ANSWER
13.	5	**E.** $16\left(\sqrt{2}-1\right)\pi$
14.	5	**E.** $\frac{1}{\sqrt{2}}$
15.	5	**B.** 
16.	5	**B.** $\frac{1}{5}\int_C (4y\,dx - x\,dy)$
17.	5	**E.** -2
18.	5	**B.** 14
19.	5	**D.** only (i) and (iii)
20.	5	**B.** 72π
21.	5	**C.** 32/5