Problem 1 (0 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 120 minutes, put away your notes and calculator, and solve the exam in one sitting.

The real Final Exam will have 20 questions with each question worth 5 points. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded)

Problem 2 (5 points)

A line l passes through the points $A\,(1,-2,1)$ and $B\,(2,3,-1)$. At what point does this line intersect with the xy-plane?

To receive the full 5 points, you must show all your work on this problem.

A. $\left(\frac{3}{2},\frac{1}{2},0\right)$

B. $\left(\frac{3}{2},-1,0\right)$

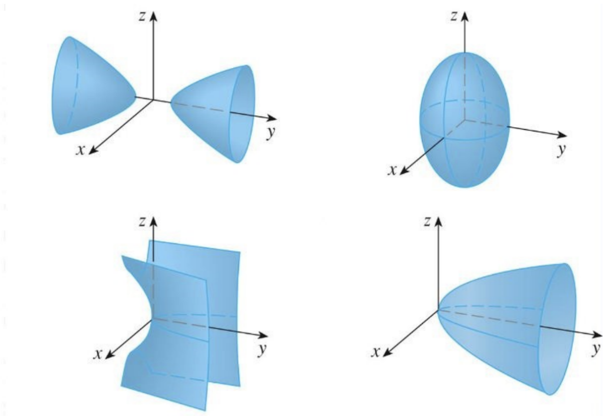
C. $\left(\frac{5}{2},\frac{1}{2},0\right)$

D. $\left(\frac{3}{2},\frac{-1}{2},0\right)$

E. $\left(\frac{5}{2},\frac{-1}{2},0\right)$

Problem 3 (5 points)

Which of the following equations produces a surface that is NOT shown here?



A. $y = x^2 - z^2$

B. $9x^2 + 4y^2 + z^2 = 1$

C. $-x^2 + y^2 - z^2 = 1$

D. $x^2 - y^2 + z^2 = 1$

E. $y = 2x^2 + z^2$

Problem 4 (5 points)

If L is the tangent line to the curve $\vec{r}(t) = \langle 2t - 1, t^2, t^2 - 2 \rangle$ at (3, 4, 2), find the point where L intersects the xy −plane.

A. (1, 2, 0)

B. (0, 0, 0)

C. (2, 1, 0)

D. (2, -2, 0)

E. (2, 2, 0)

Problem 5 (5 points)

The arclength of the curve $\vec{r}(t) = 2t\vec{i} + t^2\vec{j} + (\ln t)\vec{k}$ for $1 \leq t \leq 2$ is

A. 35/5

B. $5 + \ln 2$

C. $4 + \ln 2$

D. $3 + \ln 2$

E. 5

Problem 6 (5 points)

The level curves of $f(x,y) = \sqrt{x^2 + 4y^2 + 4} - x$ are

A. parabolas
B. hyperbolas
C. ellipses
D. sometimes lines and sometimes ellipses
E. circles

Problem 7 (5 points)

Suppose that z is defined as a function of x and y by the equation

$\cos(xyz) = x + 3y + 2z.$

Use implicit differentiation to find the value of $\frac{\partial z}{\partial y}(0,1)$.

A. -2/3
B. -3/2
C. -3/5
D. -1/2
E. 1/3

Problem 8 (5 points)

Find the directional derivative of the function $f(x,y,z) = x^2y + y^2z$ at (1,2,3) in the direction toward the point (3,1,5).

A. 1
B. 3
C. -2
D. 1/3
E. -1

Problem 9 (5 points)

Consider the function

$$f(x,y) = \frac{1}{4}x^4 + xy + \frac{1}{4}y^4 \text{ on } R^2$$

Then the function

A. has one saddle point and two local minima.
B. has one local maximum and two local minima.
C. has an absolute maximum and absolute minimum.
D. has 4 critical points.
E. is always positive and hence has absolute minimum of 0.

Problem 10 (5 points)

The extreme values of $f(x,y,z) = 3x + 2y + 6z$ with constraint $x^2 + y^2 + z^2 = 4$ are.

To receive the full 5 points, you must show all your work on this problem.

A. The maximum of f is 7 and the minimum of f is -14
B. The maximum of f is 14 and the minimum of f is -14
C. The maximum of f is 7 and the minimum of f is -7
D. The maximum of f is 14 and the minimum of f is -7
E. The maximum of f is 28 and the minimum of f is -28

Problem 11 (5 points)

By changing the order of integration, compute $\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1-y^2}\, dy\, dx$

A. 0
B. $1/3$
C. $\pi/4$
D. $2/3$
E. 1

Problem 12 (5 points)

Which of the following integrals represents the volume of the solid in the first octant

that is bounded on the side by the surface $x^2+y^2=4$ and on the top by the surface $x^2+y^2+z=4$?

A. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-z} dzdydx$

B. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} dzdydx$

C. $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{x^2+y^2} dzdydx$

D. $\int_0^2 \int_0^2 \int_0^{4-x^2-y^2} dzdydx$

E. $\int_0^4 \int_0^4 \int_0^{4-x^2-y^2} dzdydx$

Problem 13 (5 points)

convert the integral to cylindrical, then evaluate it:

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} 15\sqrt{x^2+y^2} dzdydx.$$

A. 16π

B. 64π

C. 32π

D. 43π

E. 4π

Problem 14 (5 points)

Compute the line integral $\int_C (4x^3+y^3)ds$, where C is the line segment from (0,0) to (1,2).

A. $3\sqrt{5}$

B. $\sqrt{5}\pi$

C. 0

D. -5

E. $\frac{5\sqrt{5}}{4}$

Problem 15 (5 points)

Compute the line integral

$$\int_C F \cdot dr$$

Where $F =< xy, x + y >$ and C is the curve $y = x^2$ from (0,0) to (1,1).

A. 13/12

B. 23/12

C. 5/12

D. 17/12

E. 21/12

Problem 16 (5 points)

Compute $\int_C y^2 dx + x dy$, where the curve C is the boundary of the half-disc

$$R = \{(x, y) : x^2 + y^2 \leq 9 \text{ and } x \geq 0\}$$

with clockwise orientation.

A. 0

B. 9π

C. $\frac{9\pi}{2}$

D. -3π

E. $\frac{-9\pi}{2}$

Problem 17 (5 points)

Given a two-dimensional vector field $F(x, y) = \left\langle x^2 + \frac{y}{x^2 + y^2}, x - \frac{x}{x^2 + y^2} \right\rangle$, compute the value of the scalar curl of $F(x, y)$ at the point (2,1).

A. $\frac{5}{\sqrt{5}}$

B. 1

C. $\frac{4}{\sqrt{5}}$

D. 3

E. $\frac{7}{\sqrt{5}}$

Problem 18 (5 points)

Find the surface area of the parametric surface

$$r(u,v) = \langle 2u + 3v, 3u + v, 2 \rangle, \quad 0 \le u \le 2, \quad 0 \le v \le 1.$$

A. 12

B. 4

C. $3\sqrt{2}$

D. 14

E. $4\sqrt{2}$

Problem 19 (5 points)

Let S be the part of the plane $y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 1$ Compute $\int\int_S F \cdot ndS$ for $F(x,y,z) = \langle x, 1 - y + \mathrm{e}^z, y - \mathrm{e}^z \rangle$ with S oriented in the upward normal.

A. $-\pi \mathrm{e}^2$

B. π

C. $2e\pi$

D. 2π

E. $1 - 4\pi$

Problem 20 (5 points)

Evaluate the integral $\iint_S curl\vec{F} \cdot dS$ using Stoke's Theorem, where $\vec{F} = -y\vec{i} + x\vec{j} + xyz\vec{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the xy-planes, oriented upward.

A. 4π

B. 2π

C. 0

D. 8π

E. -8π

Problem 21 (5 points)

Compute

$$\iint_S F \cdot n dS$$

the net outward flux of the vector field $F = \langle x + y, y - z, xy + z \rangle$ across the surface S, which is the boundary of the solid bounded by $z = 0, y = 0, y + z = 2$, and $x = 1 - x^2$.

A. −32/5

B. 32/15

C. −32/15

D. −16/5

E. 32/5

Answer Key

PROBLEM	POINTS	ANSWER
1.	0	$Name$
2.	5	**A.** $\left(\frac{3}{2},\frac{1}{2},0\right)$
3.	5	**D.** $x^2 - y^2 + z^2 = 1$
4.	5	**E.** (2, 2, 0)
5.	5	**D.** $3 + \ln 2$
6.	5	**A.** parabolas
7.	5	**B.** -3/2
8.	5	**A.** 1
9.	5	**A.** has one saddle point and two local minima.
10.	5	**B.** The maximum of f is 14 and the minimum of f is -14
11.	5	**D.** $2/3$
12.	5	**B.** $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} dzdydx$

PROBLEM	POINTS	ANSWER
13.	5	**B.** 64π
14.	5	**A.** $3\sqrt{5}$
15.	5	**D.** 17/12
16.	5	**E.** $\frac{-9\pi}{2}$
17.	5	**B.** 1
18.	5	**D.** 14
19.	5	**B.** π
20.	5	**D.** 8π
21.	5	**E.** 32/5