Exam 1 Study Guide

MA 261 · Fall 2023

Exam 1 is Tuesday, Oct 3 at 8:00pm – 9:00pm.

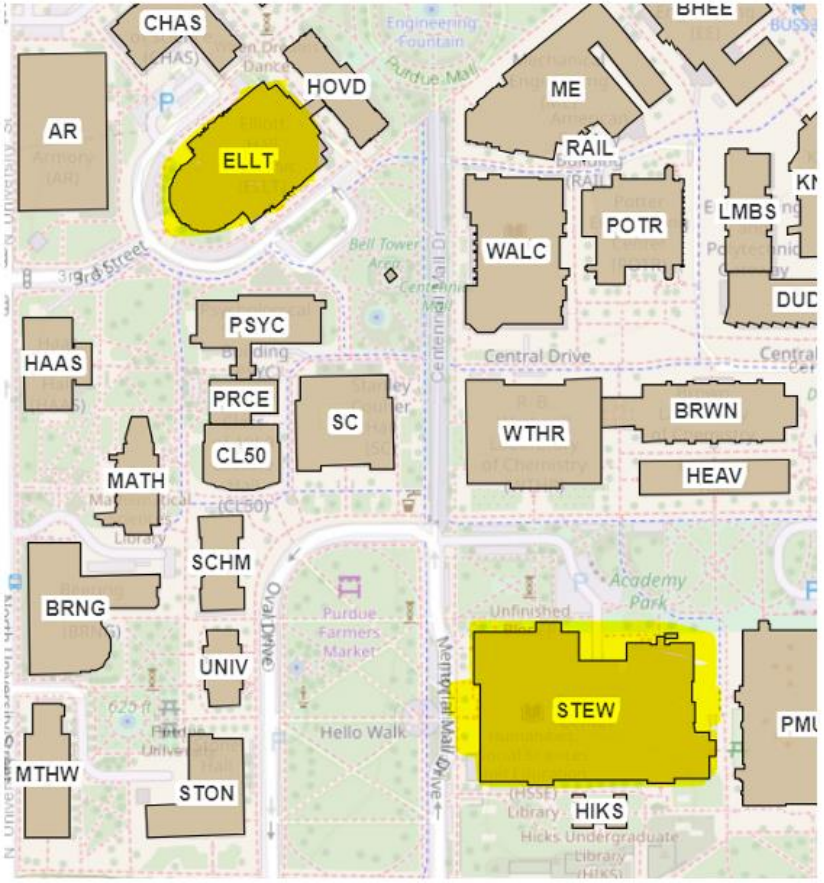
Exam Format: The exam will be in-person. You will have 1 hour (60 minutes) to complete the exam. It is a multiple-choice exam with a total of 12 questions. Each question is worth 8 points, and you will earn 4 points for filling out the scantron correctly.

Exam Material: The exam will cover Lessons 1 – 16 (Chapters 13-14, and 15.1-15.7). Exam 1 will not cover Section 15.8 Lagrange Multipliers. A detailed list of learning objectives, definitions, and practice problems is included in the table below.

Past Exam Archive:
https://www.math.purdue.edu/academic/courses/oldexams.php?course=MA26100

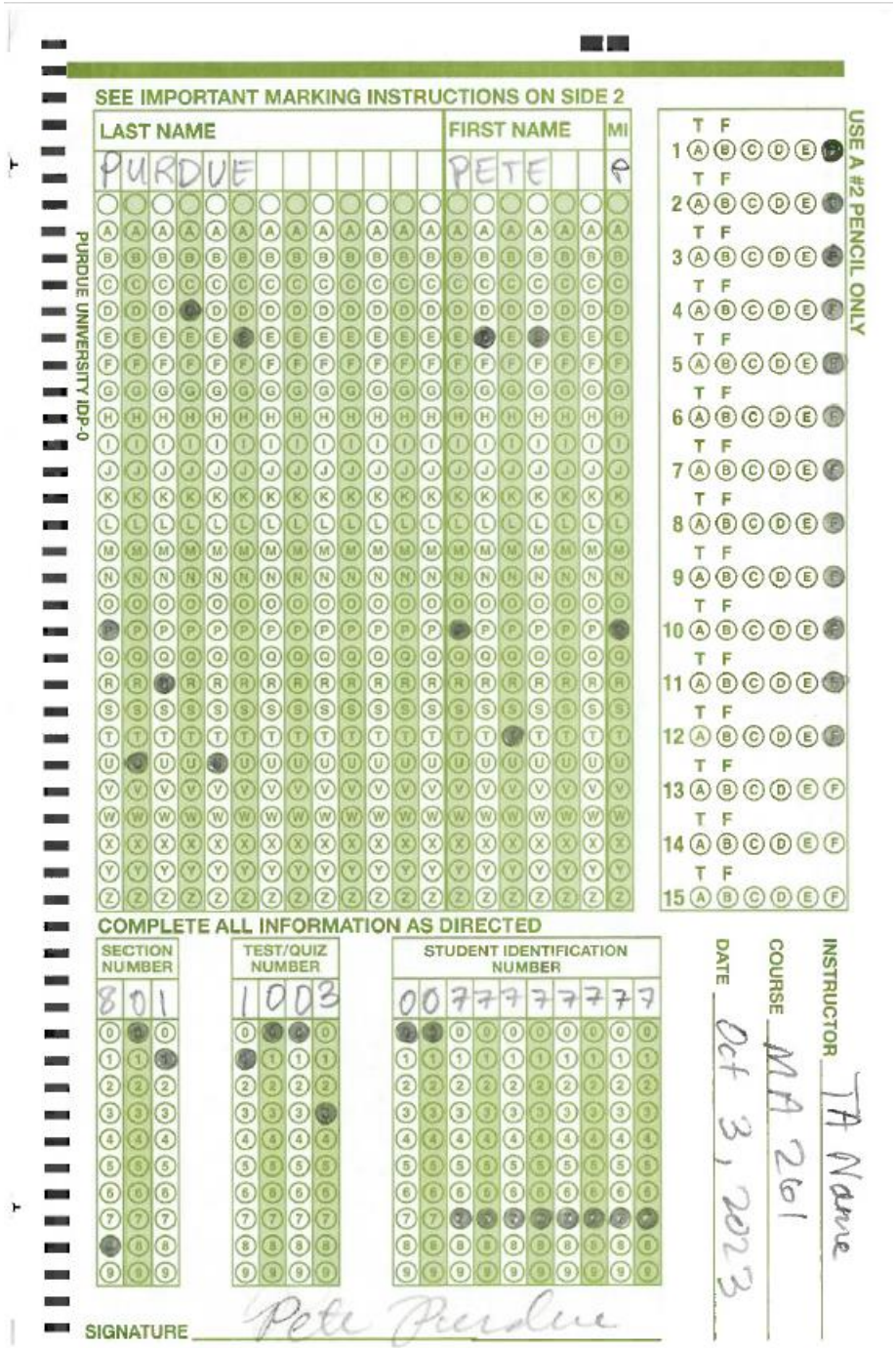
Exam Conflicts: If a student has a conflict with the evening exam, they must provide documentation with the Exam Conflict Form: https://www.math.purdue.edu/~kthood/docs/MA261_Fall2023/exam_conflict_form_ma261_fa23.pdf

ADA Accommodated Exams: Most students are automatically scheduled for their ADA exam. You should have received an email with details (and you may need to check your spam folder). For those students who were not automatically enrolled, Dr. Hood has sent you an email with further instructions.



Exam Location: The exam will be in ELLT 116 and in STEW 183 (Loeb Playhouse). You will be assigned a room with your TA. All the students who have the same TA will sit in the same area of the exam room. There should be empty seats to your left and right, and a person in the seat in front of you and the seat behind you.

You can view your room assignment here:
https://www.math.purdue.edu/academic/courses/semester/202410/ma26100/resources/ma261_exam_seating_chart.pdf



Exam Rules:

Test Booklet: There will be a scantron (left) and a test booklet. Students may not open the test booklet until instructed.

Test/Quiz Number: There will be ten versions of the exam, each with a 4-digit "Test/Quiz Number". The student must put the "Test/Quiz Number" on the scantron in order for their scantron to be correctly graded.

Please Bring: A number 2 pencil, your PUID, your REC section number, and your TA's name.

Prohibited Items: Notes, Textbooks, Calculator, Phones, Smart Watches, and all other Electronic Devices. You are not allowed to communicate to anyone else during the exam except, if you have a question, your TA or lecturer.

If a student uses one of the prohibited items during the exam, it will be considered a violation of the academic honesty policy and reported to the Office of the Dean of Students. All electronic devices should be turned off and put away out of sight.

Late Policy: Students must arrive promptly to take the exam. If a student arrives more than 20 minutes late, they will not be permitted to take the exam. Instead, the student will have to take a make-up exam with a 20% late penalty. Additionally, if a student finishes early, they cannot leave the exam room until after 20 minutes have passed.

Leaving Early Policy: In the last 10 minutes, students may not leave the exam room. This is to minimize noise and distractions for the other students still working on the exam. The student may raise their hand and turn in the exam to the TA or proctor, but they must remain seated.

#	Lesson:	Sec:	Quiz:	Learning Objectives: (You should be able to)	Definitions:	Textbook Problems:	Past Exam Problems:
1	Review of Vectors	13.1 13.2 13.3 13.4	1	- Compute vector operations - Find magnitude of a vector - Find a position vector - Find the equation of a sphere, ball, or circle - Find equations of simple planes - Compute dot products - Find angles between vectors - Calculate orthogonal projections - Compute cross products - Find areas of parallelograms and triangles - Find orthogonal vectors	Vector, scalar, magnitude, zero vector, position vector, unit vector, parallel, sphere, ball, circle, plane, dot product, orthogonal, orthogonal projection, cross product, determinant, coordinate unit vectors (**i**, **j**, **k**)	**13.1:** # 21, 23, 29, 43, 45, 47 **13.2:** # 31, 37, 41, 45, 71, 73 **13.3:** # 25, 27, 35, 37, 39, 41, 43, 47, 53, 55, 63, 65 **13.4:** # 13, 21, 25, 27, 29, 33, 43, 45, 49	Few exam questions test these concepts directly. However, you will need these concepts to complete the more difficult questions from the past exams. S18E1#3 S18FE#1 S16E1#1
2	Lines & Planes in Space	13.5	1	- Find equations of lines and line segments - Find equations of planes - Determine whether planes are parallel, intersecting, or identical - Find intersections between lines and/or planes	Parallel, intersecting, skew, orthogonal planes	**13.5**: # 21, 23, 25, 49, 51, 55, 57, 65, 67, 69, 73, 75, 77, 79, 81, 90	S19E1#1 S19E1#2 S19FE#1 F19E1#1 F19FE#1 F18E1#1 F18FE#1
3 4	Quadratic Surfaces	13.6	2	- Sketch graphs of cylinders and quadratic surfaces - Identify surfaces from equations	Trace, elliptic paraboloid, ellipsoid, cylinder, elliptic cone, hyperboloid of one sheet, hyperboloid of two sheets, hyperbolic paraboloid	**13.6**: # 15, 17, 19, 21, 23, 25, 27, 31, 35, 37, 39, 43, 45, 47, 49, 51, 55, 57, 59	S19FE#2 F19E1#2 S18E1#1 F18E1#3 F18FE#2
5	Vector-Valued Functions	14.1	2	- Graph curves described by vector-valued functions - Find domains of vector-valued functions - Find the intersection of planes and curves defined by vector-valued functions	Vector-valued function, domain, limit of a vector-valued function	**14.1**: # 11, 12, 15, 17, 19, 21, 39, 41, 43, 45, 47, 49, 53, 55	S22E1#4 S19E1#3 F19FE#2 F18E1#2 F16E1#4 S14E1#9

#	Lesson:	Sec:	Quiz:	Learning Objectives: (You should be able to)	Definitions:	Textbook Problems:	Past Exam Problems:
6	Calculus of Vector-Valued Functions, Motion in Space	14.2 14.3	3	- Find first derivatives of vector-valued functions - Find tangent vectors and tangent lines for vector-valued functions - Evaluate definite integrals of vector-valued functions - Find velocity, speed, and acceleration of objects	Tangent vector, unit tangent vector, tangent line, derivative rules	**14.2**: # 11, 13, 15, 17, 19, 21, 25, 27, 29, 35, 37, 39, 47, 71, 73, 75, 77, 79, 81, 83 **14.3**: # 47, 49	S18E1#2 S18E1#3 S17E1#3 S16E1#5
7	Motion in Space	14.3	3	- Compare trajectories of objects - Solve applications involving 2d and 3d motion	Velocity, acceleration, trajectories	**14.3**: # 15, 17, 19, 29, 31, 47, 49, 57	S19E1#6 S19FE#20 F19E1#3 F19E1#6 S18FE#3 F18E1#5 F18E1#6
8	Length of Curves, Curvature	14.4 14.5	3	- Find arc lengths of vector-valued functions - Parameterize curves by arc length - Find unit tangent vectors and curvatures - Use velocity to find curvature	Arc length, curvature	**14.4**: # 9, 11, 13, 15, 17, 19, 23, 25, 33, 35, 37 **14.5**: # 11, 13, 15, 17, 21, 23, 25	**Arc Length** S19E1#5 F19E1#5 F19FE#3 S18FE#2 **Curvature** S19E1#4 F19E1#4 F18E1#4
9	Functions of Several Variables	15.1	4	- Find domains of functions - Graph surfaces - Graph level curves of functions	Function of several variables, level curves	**15.1**: # 15, 19, 21, 23, 27, 29, 31, 33, 35, 37, 39, 41, 57	S19E1#7 S18E1#5 F18E1#7
10	Limits and Continuity	15.2	4	- Evaluate limits of functions - Evaluate limits at boundary points - Determine where functions are continuous	Limit laws, boundary point, interior point, two-path test, continuity	**15.2**: # 17, 19, 21, 23, 25, 27, 29, 31, 33, 39, 41, 49, 53, 61, 63, 65, 67, 69, 71, 77	F19E1#7 F18E1#8 S17E1#6
11	Partial Derivatives	15.3	4	- Find first partial derivatives - Find second partial derivatives	Partial derivative, differentiable,	**15.3**: # 17, 19, 21, 27, 33, 39, 41, 43, 45, 47, 55, 57, 79	S19E1#8 S19FE#7 F19E1#8 F19FE#6

#	Lesson:	Sec:	Quiz:	Learning Objectives: (You should be able to)	Definitions:	Textbook Problems:	Past Exam Problems:
12	The Chain Rule	15.4	5	- Use the chain rule to find derivatives - Differentiate implicitly - Evaluate partial derivatives at specified points	Chain rule, implicit differentiation	**15.4**: # 15, 17, 19, 21, 23, 35, 37, 39,	S19E1#9 F19FE#7 S18FE#5 F18E1#10 F18FE#5
13	Directional Derivatives and the Gradient	15.5	5	- Compute gradients and/or directional derivatives - Find directions or paths of change - Compute slopes of lines tangent to level curves	Gradient, directional derivative, directions of change, level curves, steepest descent	**15.5**: #17, 19, 25, 27, 29, 31, 33, 35, 61, 63, 65, 67	S19E1#10 S19FE#8 F19E1#9 F19FE#4 S18FE#7 F18E1#11 F18FE#4
14	Tangent Plane and Linear Approximation	15.6	5	- Find equations of planes tangent to surfaces - Find linear approximations - Use differentials to approximate changes in functions	Tangent plane, differential, linear approximation	**15.6**: #13, 15, 17, 19, 21, 25, 29, 33, 35, 37, 53	S19E1#11 S19FE#5 F19FE#5 S18FE#6 F18E1#9 F18FE#3
15 16	Maximum and Minimum Problems	15.7	-	- Find and analyze critical points for functions - Find local and absolute extrema for functions	Local extrema, critical point, saddle point, second derivative test, absolute extrema	**15.7**: #9, 11, 25, 27, 29, 31, 33, 35, 37, 39, 41, 47, 49, 51, 53, 55, 63, 67	S19E1#12 S19FE#9 F19E1#10 F19E1#11 F19FE#8 S18FE#9 F18FE#7 F18E1#12 F18E2#1

*Answers to the practice problems are included in the textbook starting on page A-61.

Sample Exam Booklet Cover Sheet

MA 26100
Exam 1
10/03/2023
TEST/QUIZ NUMBER:
1003

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION # _____

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough spaces for your name, fill in as much as you can.
2. Your 3-digit recitation section number, e.g. **XYZ**. (If you don't know your recitation section number, ask your TA.)
3. Test/Quiz number: **1003**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros**
5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–12.

There are **12** questions, each worth 8 points (you will earn 4 points for filling out your scantron correctly). Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 8:50pm, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20pm. If you don't finish before 8:50pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

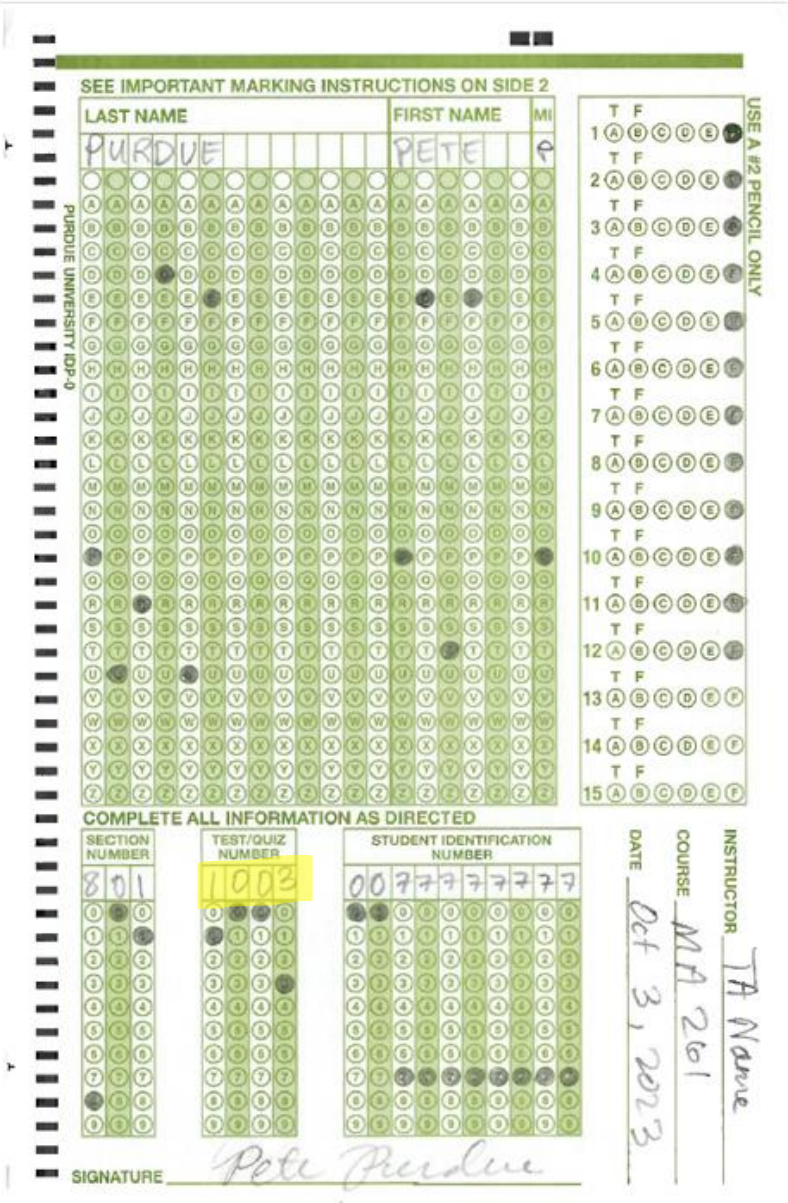
EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, phone, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: _____

Sample Scantron



Problem 1 (4 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 60 minutes, put away your notes and calculator, and solve the exam in one sitting.

On the real exam, you will receive 4 points for correctly filling out the scantron. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded)

Answer = _____

Problem 2 (8 points)

Find an equation of the plane that contains the point (1, 2, −3) and the line with symmetric equations $x - 2 = y - 1 = \dfrac{z + 2}{2}$.

To receive the full 5 points, you must show all your work on this problem.

A. $5x + y + z = 4$

B. $4x - 2y - 3z = 9$

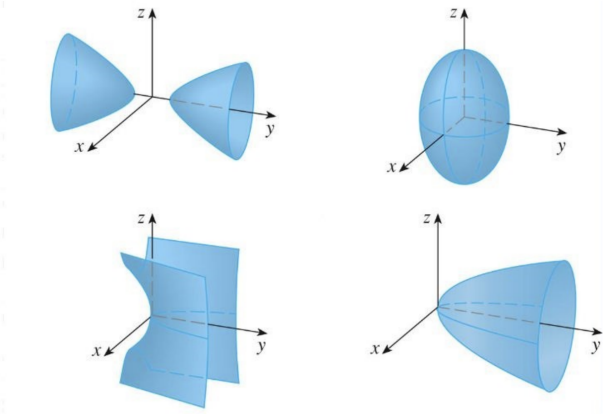
C. $x + y - 2z = 9$

D. $3x + y - 2z = 11$

E. $2x - y + z = -3$

Problem 3 (8 points)

Which of the following equations produces a surface that is NOT shown here?



A. $y = x^2 - z^2$

B. $x^2 - y^2 + z^2 = 1$

C. $y = 2x^2 + z^2$

D. $9x^2 + 4y^2 + z^2 = 1$

E. $-x^2 + y^2 - z^2 = 1$

Problem 4 (8 points)

The domain of the vector function $r\,(t) = \left\langle \sqrt{t^2 - 4t + 3}, e^{3t}, \ln\left(t^{\frac{1}{3}} - 1\right) \right\rangle$

To receive the full 5 points, you must show all your work on this problem.

A. t is any real number

B. $t \geq 3$

C. $1 < t < 3$

D. $t > 1$

E. None of the above

Problem 5 (8 points)

A particle has acceleration $a = \left\langle 6t - 2, \frac{-1}{t^2}, 0 \right\rangle$. It is known that the velocity at time $t = 1$ is $v(1) = \langle 1,1,1 \rangle$ and that the position vector at time $t = 1$ is $r(1) = \langle 0,0,3 \rangle$. Find the magnitude of the position vector at time $t = 2$.

To receive the full 5 points, you must show all your work on this problem.

A. $\sqrt{16 + \ln 4}$
B. $\sqrt{32 + (\ln 2)^2}$
C. $\sqrt{16 + (\ln 2)^2}$
D. $\sqrt{32 + (\ln 4)^2}$
E. 4

Problem 6 (8 points)

If $\vec{r}(t) = \langle 1, 5t^2, 4t \rangle$, find $\kappa(0)$ (i.e., the curvature at $t = 0$).

A. 1
B. -5/4
C. 0
D. 5/8
E. 5/4

Problem 7 (8 points)

The level curves of $f(x,y) = \sqrt{x^2 + y^2 + 1} + x$ are

A. circles
B. sometimes lines sometimes ellipses
C. ellipses
D. parabolas
E. hyperbolas

Problem 8 (8 points)

Consider the limits

$$A = \lim_{(x,y)\to(0,0)} \frac{3x-2y}{\sqrt{x^2+y^2}} \text{ and } B = \lim_{(x,y)\to(0,0)} \frac{e^{x+y}}{1+e^{x-y}}$$

Which one of the following statements is true?

A. Both A and B do not exist

B. A does not exist and B = $\frac{1}{2}$

C. A=3 and B=1

D. A = 1 and B = 1

E. A does not exist and B = 1

Problem 9 (8 points)

Let $f(x,y) = e^{x+3y-3}\sin(\pi xy)$. Find $\frac{\partial f}{\partial x}(1,1)$.

To receive the full 5 points, you must show all your work on this problem.

A. $e\pi$

B. $-e\pi$

C. $-\pi$

D. $-e$

E. $-e\pi^2$

Problem 10 (8 points)

Let $z = e^r \cos\theta, r = 12st, \theta = \sqrt{s^2 + t^2}$. The partial derivative $\frac{\partial z}{\partial s}$ is:

A. $12t\cos\theta - \frac{se^r \sin\theta}{\sqrt{s^2 + t^2}}$

B. $e^r \left(t\cos\theta - \frac{s\sin\theta}{\sqrt{s^2 + t^2}} \right)$

C. $e^r \left(t\cos\theta + \frac{s\sin\theta}{\sqrt{s^2 + t^2}} \right)$

D. $e^r \left(12t\cos\theta + \frac{s\sin\theta}{\sqrt{s^2 + t^2}} \right)$

E. $e^r \left(12t\cos\theta - \frac{s\sin\theta}{\sqrt{s^2 + t^2}} \right)$

Problem 11 (8 points)

Find the directional derivative of $f(x,y) = xe^{y^2} + e^{x+y}$ at the point (0,0) in the direction of the vector $3\vec{i} - 4\vec{j}$.

To receive the full 5 points, you must show all your work on this problem.

A. 6/5
B. -6/5
C. 0
D. -2/5
E. 2/5

Problem 12 (8 points)

Find a so that the point $(3,a,1)$ is on the tangent plane to $z = e^{xy} - 4x^2y + 3y^2$ at (0,1,4).

To receive the full 5 points, you must show all your work on this problem.

A. 1/2
B. 1/6
C. -1/7
D. -1/2
E. 0

Problem 13 (8 points)

Classify the critical points (2, 2) and (-3, 0) of $g(x,y)$ if

$g_x\,(2,2)= 0, g_y\,(2,2)= 0, g_{xx}\,(2,2)= -2, g_{yy}\,(2,2)= -2, g_{xy}\,(2,2)= -1$

$g_x\,(-3,0)= 0, g_y\,(-3,0)= 0, g_{xx}\,(-3,0)= 0, g_{yy}\,(-3,0)= -6, g_{xy}\,(-3,0)= -3$

A. A local minimum at (2, 2) and a local maximum at (−3, 0)
B. A local minimum at (2, 2) and a saddle point at (−3, 0)
C. A local maximum at (2, 2) and a local minimum at (−3, 0)
D. A local maximum at (2, 2) and a saddle point at (−3, 0)
E. A saddle point at (2, 2) and a local minimum at (−3, 0)

Answer Key

PROBLEM	ANSWER
1.	$Name$
2.	**D.** $3x + y - 2z = 11$
3.	**B.** $x^2 - y^2 + z^2 = 1$
4.	**B.** $t \geq 3$
5.	**B.** $\sqrt{32 + (\ln 2)^2}$
6.	**D.** 5/8
7.	**D.** parabolas
8.	**B.** A does not exist and B = $\frac{1}{2}$
9.	**B.** $-e\pi$
10.	**E.** $e^r \left(12t \cos\theta - \frac{s \sin\theta}{\sqrt{s^2 + t^2}} \right)$
11.	**E.** 2/5
12.	**E.** 0

PROBLEM	ANSWER
13.	**D.** A local maximum at (2, 2) and a saddle point at (−3, 0)

Problem 1 (4 points)

This practice exam is provided as a way for students to practice solving the exam under a time limit. We suggest that you set a timer for 60 minutes, put away your notes and calculator, and solve the exam in one sitting.

On the real exam, you will receive 4 points for correctly filling out the scantron. The real exam is multiple choice graded all-or-nothing. (In this practice exam, ignore any prompts to show your work. While showing your work is good practice, on the exam it will not be graded)

Answer = _____

Problem 2 (8 points)

Which of the following pairs of planes are orthogonal to each other?

To receive the full 5 points, you must show all your work on this problem.

A. $x = 5z + 3y, 8x - 6y + 2z = -1$

B. $x + 10y - z = 6, -9x - y - 19z = 2$

C. $5x + 8y = -3, y + 6z = 1$

D. $8x + 5y = -3, y + 6z = -1$

E. $8x + 5y = -3, 9y + 6z = -1$

Problem 3 (8 points)

Identify the surface defined by $x^2 - y^2 - 4x + z^2 = 4$.

A. ellipsoid
B. hyperboloid of one sheet
C. hyperbolic paraboloid
D. cone
E. hyperboloid of two sheets

Problem 4 (8 points)

The domain of the vector function $r(t) = \left\langle \sqrt{t^2 - 4t + 3}, e^{3t}, \ln\left(t^{\frac{1}{3}} - 1\right) \right\rangle$

To receive the full 5 points, you must show all your work on this problem.

A. None of the above

B. $t \geq 3$

C. $t > 1$

D. $1 < t < 3$

E. t is any real number

Problem 5 (8 points)

A traveling particle has position vector at time t given by $\vec{r}(t) = \langle t\cos t, t\sin t, 9 - t^2 \rangle$. Find its speed at $t = 1$.

To receive the full 5 points, you must show all your work on this problem.

A. $\sqrt{2\pi}$
B. $\sqrt{6}$
C. 3π
D. $\tan(1)$
E. 5

Problem 6 (8 points)

Let $r(t) = \langle t, \frac{1}{2}t^2, \frac{1}{3}t^3 \rangle$, find $\kappa(1)$ (namely, the curvature at t=1).

A. -1/3
B. 1
C. $\frac{\sqrt{3}}{3}$
D. 1/3
E. $\frac{\sqrt{2}}{3}$

Problem 7 (8 points)

The level curves of $f(x,y) = \sqrt{x^2 + 4y^2 + 4} - x$ are

A. hyperbolas
B. ellipses
C. sometimes lines and sometimes ellipses
D. circles
E. parabolas

Problem 8 (8 points)

If

$$\lim_{(x,y)\to\,(0,0)} \frac{x^4 - 3a\left(x^2+y^2\right) - y^4}{x^2+y^2} = 12$$

then the number a must be equal to

A. 6
B. 12
C. 3
D. 4
E. -4

Problem 9 (8 points)

If $f(x,y) = \ln\left(x^2+y^4+2\right)$, compute $f_{xy}\,(2,1)$.

To receive the full 5 points, you must show all your work on this problem.

A. -10/49
B. 12/49
C. -16/49
D. -4/7
E. 4/7

Problem 10 (8 points)

Suppose that z is defined as a function of x and y by the equation

$\cos(xyz) = x + 3y + 2z$.

Use implicit differentiation to find the value of $\dfrac{\partial z}{\partial y}\,(0,1)$.

A. 1/3
B. -3/5
C. -1/2
D. -3/2
E. -2/3

Problem 11 (8 points)

Find the maximum rate of change of $f(x,y) = \sqrt{7 - x^2 - y^2}$ at the point (-2, 1).

To receive the full 5 points, you must show all your work on this problem.

A. $\frac{5}{\sqrt{2}}$
B. $\frac{\sqrt{10}}{2}$
C. $\sqrt{8}$
D. $\frac{3}{\sqrt{2}}$
E. 1/4

Problem 12 (8 points)

Find a so that the point $(3,a,1)$ is on the tangent plane to $z = e^{xy} - 4x^2y + 3y^2$ at (0,1,4).

To receive the full 5 points, you must show all your work on this problem.

A. -1/7
B. 0
C. 1/2
D. -1/2
E. 1/6

Problem 13 (8 points)

Consider the function

$f(x,y) = \frac{1}{4}x^4 + xy + \frac{1}{4}y^4$ on R^2

Then the function

A. has one local maximum and two local minima.
B. has 4 critical points.
C. has one saddle point and two local minima.
D. has an absolute maximum and absolute minimum.
E. is always positive and hence has absolute minimum of 0.

Answer Key

PROBLEM	ANSWER
1.	$Name$
2.	**B.** $x + 10y - z = 6, -9x - y - 19z = 2$
3.	**B.** hyperboloid of one sheet
4.	**B.** $t \geq 3$
5.	**B.** $\sqrt{6}$
6.	**E.** $\frac{\sqrt{2}}{3}$
7.	**E.** parabolas
8.	**E.** -4
9.	**C.** -16/49
10.	**D.** -3/2
11.	**B.** $\frac{\sqrt{10}}{2}$
12.	**B.** 0

PROBLEM	ANSWER
13.	**C.** has one saddle point and two local minima.