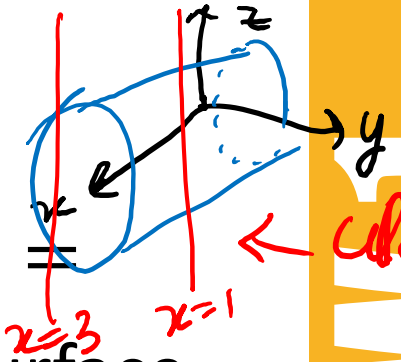
FINAL EXAM REVIEW 1

MA 26100-FALL 2023

DR. HOOD

WARM UP

(Spring 2019 Final Exam #19) Divergence Thm



Evaluate the flux integral $\iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, dS$ where $\vec{\boldsymbol{F}}(x,y,z) = \langle 3xy^2, x\cos(z), z^3 \rangle$ and S is the complete boundary surface of the solid region bounded by the cylinder $y^2 + z^2 = 2$ and the planes $x = 1$ and $x = 3$ with outward pointing normal.

a) 9π

b) 12π

c) 14π

d) 18π

e) 24π

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_E \vec{\nabla}\cdot\vec{F}\,dV$$

$$\vec{\nabla}\cdot\vec{F} = 3y^2 + 3z^2$$

$$= \int_1^3 \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} (3y^2 + 3z^2)\, dz\, dy\, dx$$

polar

$$= \int_1^3 \int_0^{2\pi} \int_0^{\sqrt{2}} 3r^2 \cdot r\, dr\, d\theta\, dx$$

$$= \cdots = 12\pi$$

FINAL REVIEW!

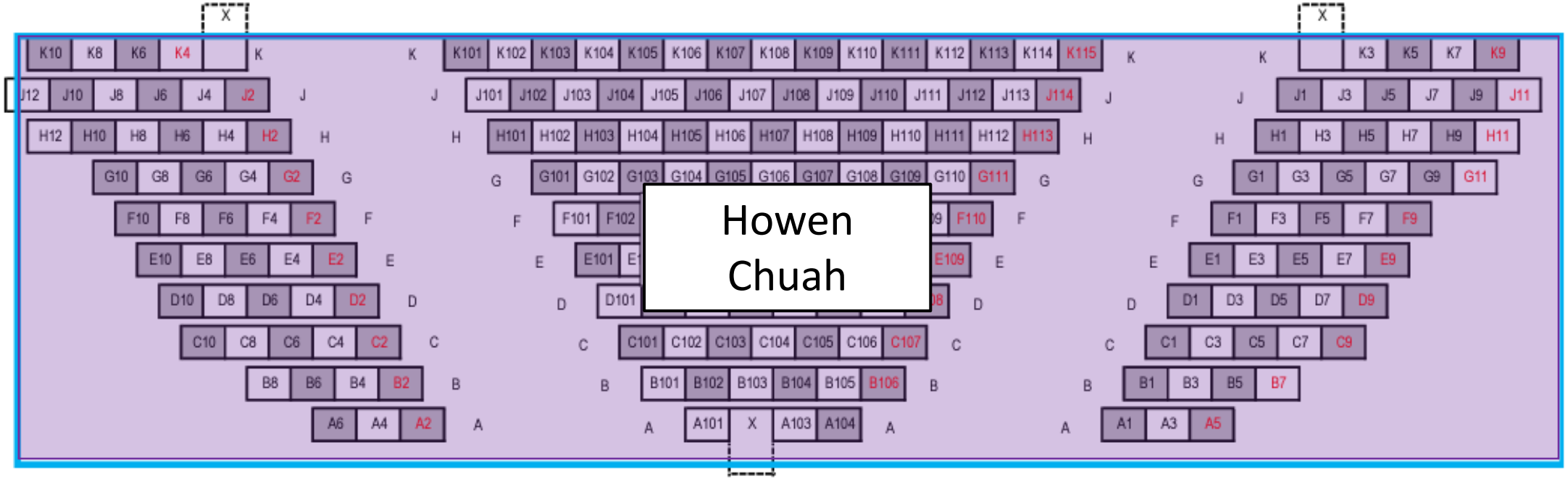
ANNOUNCEMENTS

- Final Exam is Mon Dec 11 at 8:00am- 10:00am
  - Note: in the morning!

- Please fill out course evaluations: https://purdue.evaluationkit.com/MyEval/Login.aspx

FINAL EXAM SEATING CHART

- New seating arrangements for the Final Exam:
  - 23 TAs in ELLT
  - 1 TA in RHPH

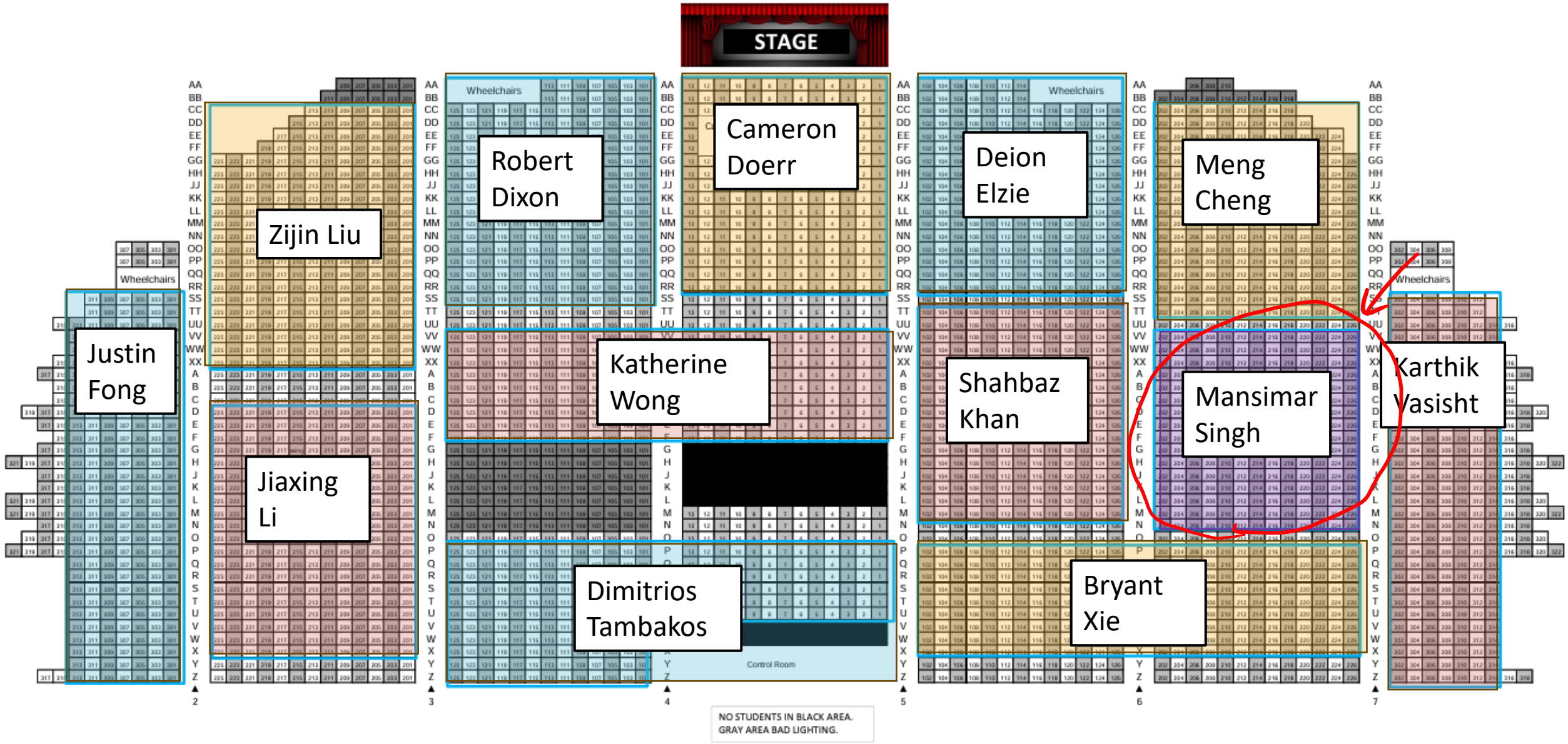


MA 26100
FINAL
Mon., Dec. 11, 2023
8:00-10:00 a.m.
RHPH 172

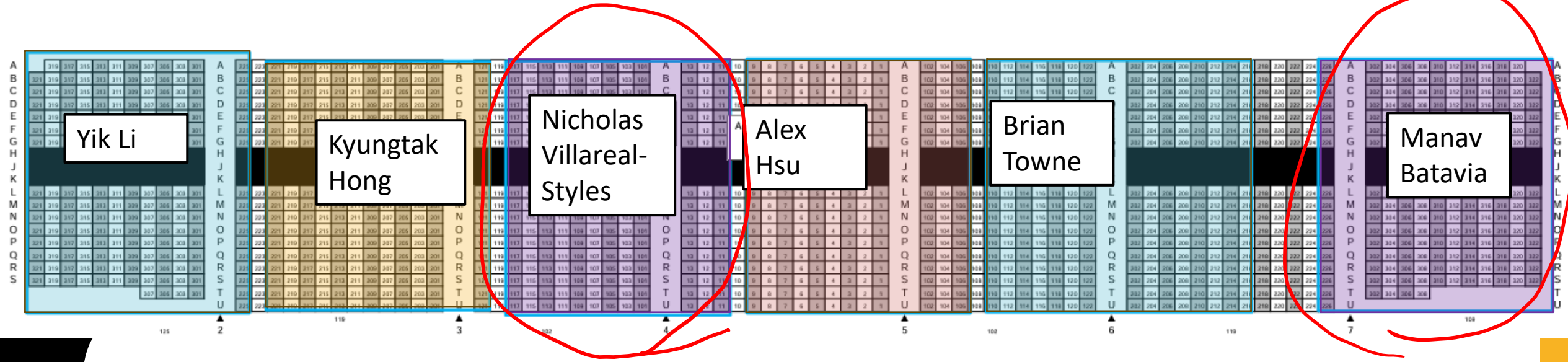
MA 26100
Final

Elliot Hall Main Floor ELLT 116

Monday, December 11, 2023
8:00 - 10:00 a.m.



MA 26100
FINAL
Mon., Dec. 11,2023
8:00-10:00 a.m.
ELLT HALL 116
FIRST BALCONY



MA 26100
FINAL
Mon., Dec. 11,2023
8:00-10:00 a.m.
ELLT HALL 116
SECOND BALCONY

EXAM 1 STATS

- **Exam 1:** Most missed questions (Test/Quiz # 1111)
  - #4 – symmetric equation of a line
  - #3 – direction of steepest descent
  - #1 – calculus of vector-valued functions

(Fall 2023 Exam 1 #4 Test/Quiz # 1111)

4. Determine a so that the line

$$\frac{x-3}{a} = \frac{y+5}{2} = \frac{z+1}{4}$$

is parallel to the plane $2x + 3y - 5z = 14$

A. 5

B. 3

C. -14

D. -4

E. 7

POLL 1

(Fall 2023 Exam 1 #3 Test/Quiz # 1111)

3. Given $f(x,y) = 25 - x^2 - y^2$, find the direction of steepest descent at the point $(x,y) = (3,-4)$.

A. $\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

B. $\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

C. $\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

D. $\left\langle \frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

E. $\left\langle -\frac{3}{5}, \frac{4}{5} \right\rangle$

steepest descent $\vec{u} = -\frac{\vec{\nabla} f}{|\nabla f|}$

$\vec{\nabla} f = \langle -2x, -2y \rangle \Big|_{\substack{x=3 \\ y=-4}}$

$= \langle -6, 8 \rangle$

$\vec{u} = \frac{\langle 6, -8 \rangle}{\sqrt{6^2 + (-8)^2}} = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$ A

(Fall 2023 Exam 1 #1 Test/Quiz # 1111)

1. A rocket travels through space with acceleration $\vec{a}(t) = \langle \cos(t), 4, e^{-t} \rangle$. At time $t = 0$ the rocket has position $\vec{r}_0 = \langle 0, 0, 0 \rangle$ and velocity $\vec{v}_0 = \langle 1, 2, 3 \rangle$. What is the position of the rocket at time $t = 2$?

A. $\langle 3 - \cos(2), 12, 7 + e^{-2} \rangle$

B. $\langle \cos(2), 8, e^{-2} \rangle$

C. $\langle -\cos(2), 8, 8 - e^{-2} \rangle$

D. $\langle 1 - \cos(2), 12, 7 + e^{-2} \rangle$

E. $\langle 1 - \cos(2), 12, 8 + e^{-2} \rangle$

EXAM 2 STATS

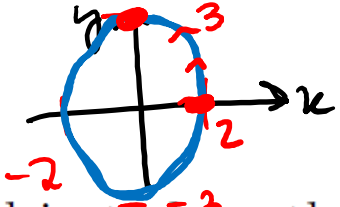
- **Exam 2:** Most missed questions (Test/Quiz # 5555)
  - #5 – vector line integral
  - #4 – fundamental theorem of line integrals
  - #1 – double integrals in polar coordinates

- Link to exam 2 booklet: https://www.math.purdue.edu/~kthood/docs/MA261_Fall2023/exam2_ver5555_ma261_fa23.pdf

POLL 2

(Fall 2023 Exam 2 #5 – Test/Quiz # 5555)

$0 \le t \le \frac{\pi}{2}$



5. Given the force field $\vec{\mathbf{F}}(x, y) = \langle -y, x \rangle$, find the work required to move an object along the ellipse $\vec{\mathbf{r}}(t) = \langle 2\cos(t), 3\sin(t) \rangle$ from $(2, 0)$ to $(0, 3)$.

A. 3π

B. 9π

C. 6π

D. 0

E. 2π

$$W = \int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\, dt$$

$$= \int_0^{\frac{\pi}{2}} \langle -3\sin(t), 2\cos(t) \rangle \cdot \langle -2\sin(t), 3\cos(t) \rangle dt$$

$$= \int_0^{\frac{\pi}{2}} 6\sin^2(t) + 6\cos^2(t)\, dt = 6 \cdot \frac{\pi}{2}$$

$$= 3\pi$$

(Fall 2023 Exam 2 #4 – Test/Quiz # 5555)

4. Find $\int_C \vec{\mathbf{F}} \cdot d\mathbf{r}$, where $\vec{\mathbf{F}}(x, y, z) = \langle 2xy - yz, x^2 - xz, -xy \rangle$ on some smooth curve C that goes from $(2, 1, 0)$ to $(3, 2, -1)$.

A. 20

B. 8

C. 12

D. 24

E. Impossible to answer without knowing C.

POLL 3

(Fall 2023 Exam 2 #1 – Test/Quiz # 5555)

1. Evaluate

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

by converting to polar coordinates.

A. 3π

B. 18π

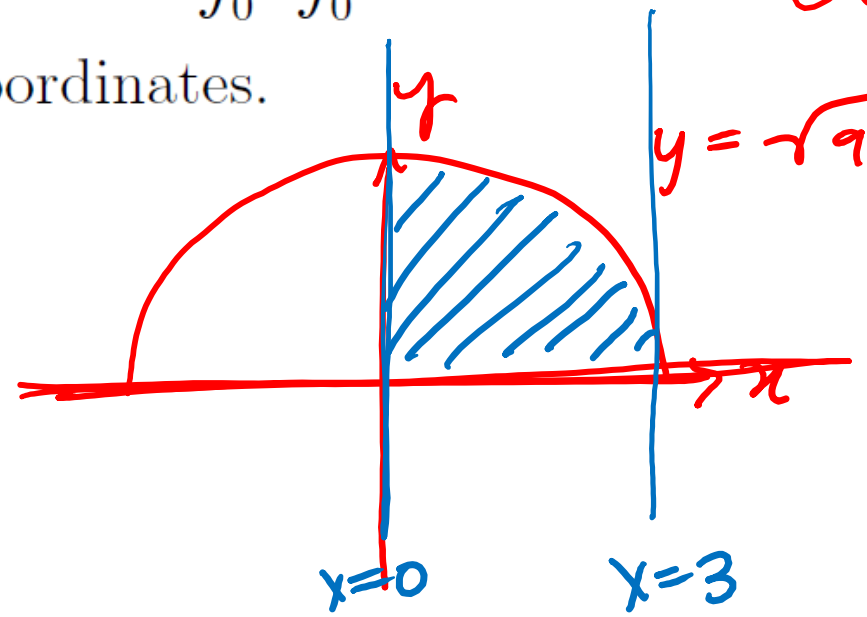
C. $\frac{9\pi}{4}$

D. 9π

E. $\frac{9\pi}{2}$

$\sqrt{x^2+y^2} \le r$

$r\,dr\,d\theta$



$y = 0$

$0 \le r \le 3$

$0 \le \theta \le \frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} \int_0^3 r \cdot r\, dr\, d\theta = \frac{9\pi}{2}$$