Exam 1 #4

$t = \frac{x-3}{a} = \frac{y+5}{2} = \frac{z+1}{4}$ is parallel to plane $2x+3y-5z=14$

$\frac{x-3}{a} = t \longrightarrow x = 3+at$

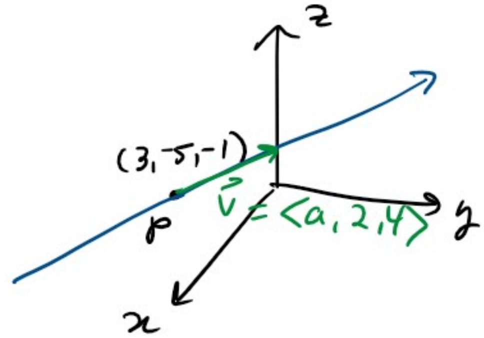
$\frac{y+5}{2} = t \longrightarrow y = -5+2t$

$\frac{z+1}{4} = t \longrightarrow z = -1+4t$

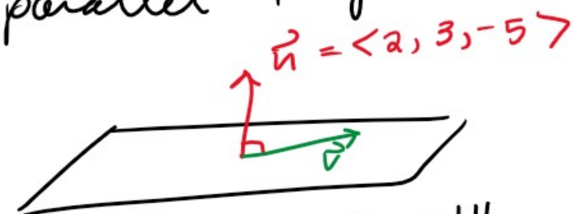
$\vec{r}(t) = \langle 3,-5,-1\rangle + t\langle a,2,4\rangle$

Direction vector

$\vec{v} = \langle a,2,4\rangle$



parallel to plane



$2x+3y-5z=14$

Want $\vec{n} \perp \vec{v}$

$\vec{n}\cdot\vec{v} = 0$

$\langle 2,3,-5\rangle \cdot \langle a,2,4\rangle = 0$

$2a+6-20=0 \rightarrow 2a = 14$ $\boxed{a=7}$ $\boxed{E}$

Exam 1 # 1

$\vec{a}(t) = \langle \cos(t), 4, e^{-t}\rangle$

@ $t=0$: $\vec{r}_0 = \langle 0,0,0\rangle$

$\vec{v}_0 = \langle 1,2,3\rangle$

What is position @ $t=2$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \int \langle \cos(t), 4, e^{-t}\rangle\,dt$

$= \langle \sin(t)+c_1,\ 4t+c_2,\ -e^{-t}+c_3\rangle$

$\vec{v}(0) = \langle 1,2,3\rangle = \langle c_1,\ c_2,\ -1+c_3\rangle$

$c_1 = 1 \quad c_2 = 2 \quad c_3 = 4$

$\vec{v}(0) = \langle 1, 2, 3 \rangle = \langle c_1,$

$c_1 = 1 \quad c_2 = 2 \quad c_3 = 4$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \int \langle \sin(t)+1,\ 4t+2,\ -e^{-t}+4 \rangle\,dt$$

$$= \langle -\cos(t) + t + d_1,\ 2t^2 + 2t + d_2,\ e^{-t} + 4t + d_3 \rangle$$

$$\vec{r}(0) = \langle 0, 0, 0 \rangle = \langle -1 + d_1,\ d_2,\ 1 + d_3 \rangle$$

$d_1 = 1 \quad d_2 = 0 \quad d_3 = -1$

$$\vec{r}(t) = \langle -\cos(t) + t + 1,\ 2t^2 + 2t,\ e^{-t} + 4t - 1 \rangle$$

$$\vec{r}(2) = \langle -\cos(2) + 3,\ 12,\ e^{-2} + 7 \rangle \qquad \boxed{A}$$

Exam 2 # 4 $\quad \vec{F} = \langle 2xy - yz,\ x^2 - xz,\ -xy \rangle$

on curve C from $(2,1,0)$ to $(3,2,-1)$

A in 3D space

$$\int_C \vec{F} \cdot d\vec{r} = \phi(B) - \phi(A) \qquad \text{if } \vec{F} = \nabla \phi$$

Fundamental Thm of Line Integrals

$$= \phi(3,2,-1) - \phi(2,1,0)$$

$$\phi = \int (2xy - yz)\,dx = x^2y - xyz + a(\cancel{y}, z)^{0}$$

$$= \int (x^2 - xz)\,dy = x^2y - xyz + b(\cancel{x}, z)^{0}$$

$$= \int (-xy)\,dz = -xyz + c(x,y)$$

$$\phi = x^2y - xyz$$

$$= \left(x^2y - xyz\right)\Big|_{\substack{x=3\\y=2}} - \left(x^2y - xyz\right)\Big|_{\substack{x=2\\y=1\\z=0}}$$

$$= \left(x^2y - xyz\right)\Big|_{\substack{x=3\\y=2\\z=-1}} - \left(x^2y - xyz\right)\Big|_{\substack{x=2\\y=1\\z=0}}$$

$$= 3^2\cdot 2 - 3\cdot 2\cdot(-1) - (2^2\cdot 1 - 0)$$

$$= 18 + 6 - 4 = 24 - 4 = \boxed{20} \quad \boxed{A}$$

If $\vec{r}(t)$ such that $\vec{r}(b) = B$, $\vec{r}(a) = A$, $a \le t \le b$

$$\Rightarrow \phi(\vec{r}(b)) - \phi(\vec{r}(a))$$