FINAL EXAM REVIEW 2

MA 26100·FALL 2023

DR. HOOD

WARM UP

For each of the following integrals, identify whether ~~its~~ ~~integrand~~ integral is a scalar or a vector. integral

a) $\int_C 2xy\,dx + 5e^x\,dy$ → vector
P, Q
$\vec{F} = \langle 2xy, 5e^x \rangle$
$d\vec{r} = \langle dx, dy \rangle$

b) $\oint_C z^2\,dS$ scalar

c) $\iint_S \left(x^2 + y^2 + z^2\right)\,dS$ scalar

d) $\iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\,dS$ where $\vec{\boldsymbol{F}}(x, y, z) = \langle 3xy^2, x\cos(z), z^3 \rangle$ vector

e) $\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ where $\vec{\boldsymbol{F}}(x, y, z) = \langle y\sin(x), z + 3, x\cos(z) \rangle$ vector

f) $\int_C \vec{\boldsymbol{\nabla}} f \cdot d\vec{\boldsymbol{r}}$ where $f(x, y, z) = \cos(xyz)$ vector
$\vec{\nabla} f = \vec{F}$ vector field

FINAL REVIEW 2

ANNOUNCEMENTS

- Extra Office Hours:
  - Thursday Dec 7 at 3:30 – 4:30pm in MATH 431

- What topics should we cover on Friday?
  - Poll: https://purdue.ca1.qualtrics.com/jfe/form/SV_8ko6p4fwSwyPxAi

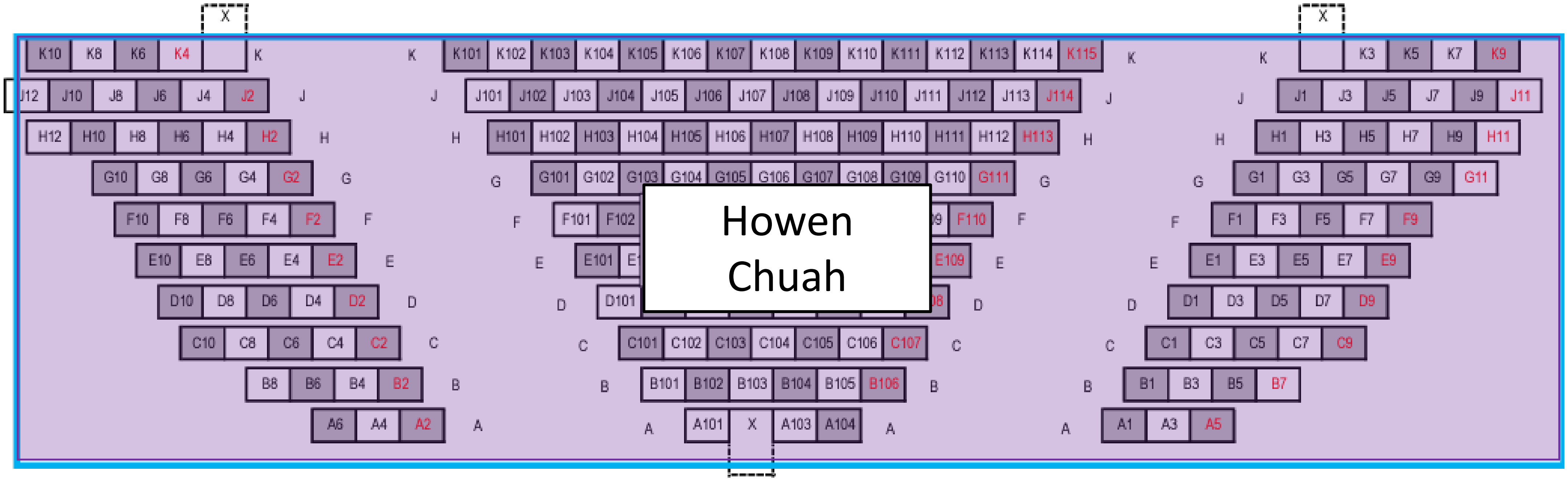
ANNOUNCEMENTS

- Final Exam is Mon Dec 11 at 8:00am- 10:00am
  - Note: in the morning!

- Please fill out course evaluations: https://purdue.evaluationkit.com/MyEval/Login.aspx

FINAL EXAM SEATING CHART

- New seating arrangements for the Final Exam:
  - 23 TAs in ELLT
  - 1 TA in RHPH (Howen Chuah)

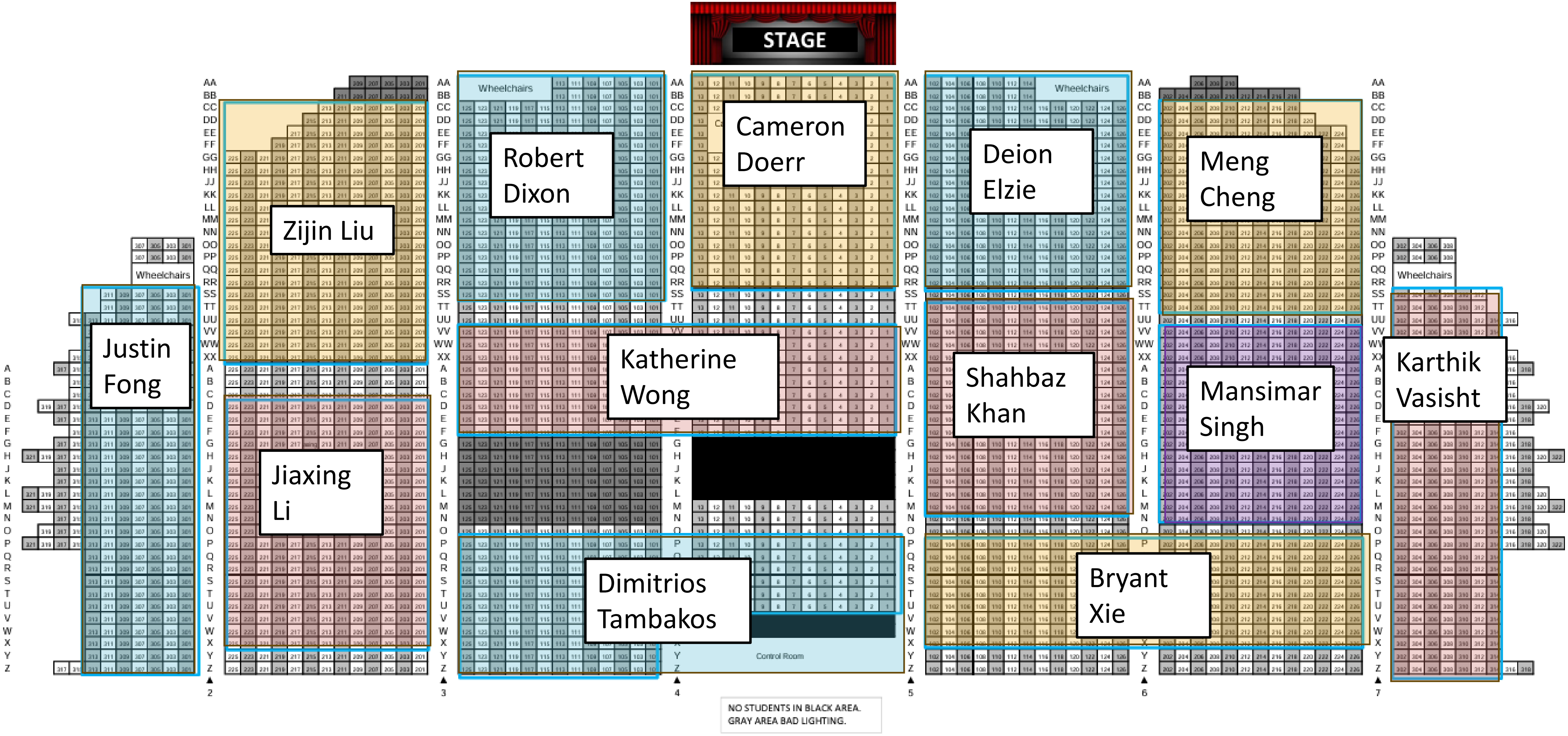


MA 26100
FINAL
Mon., Dec. 11, 2023
8:00-10:00 a.m.
RHPH 172

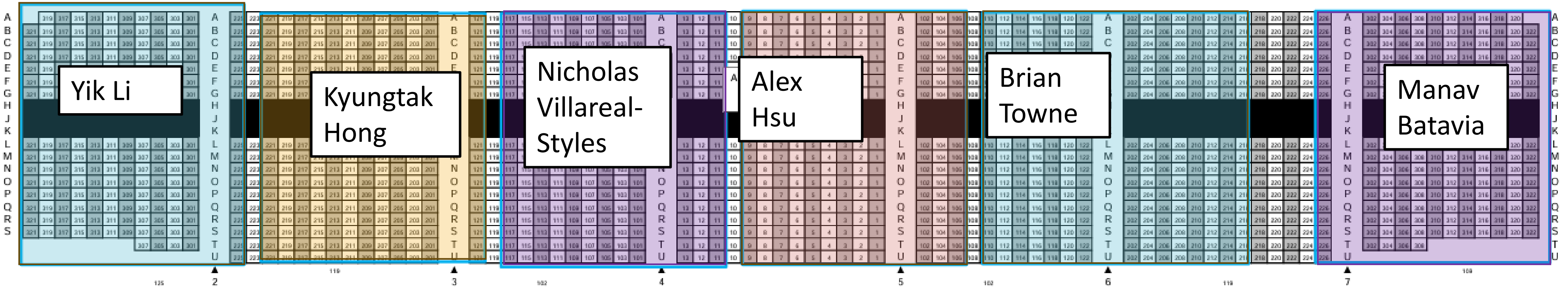
MA 26100
Final

Elliot Hall Main Floor ELLT 116

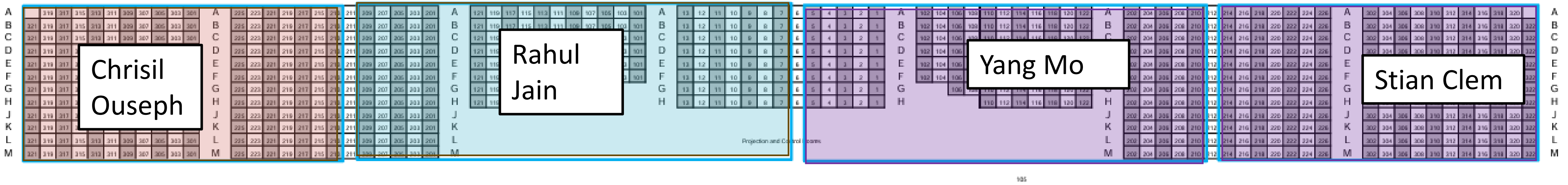
Monday, December 11, 2023
8:00 - 10:00 a.m.



MA 26100
FINAL
Mon., Dec. 11,2023
8:00-10:00 a.m.
ELLT HALL 116
FIRST BALCONY



MA 26100
FINAL
Mon., Dec. 11,2023
8:00-10:00 a.m.
ELLT HALL 116
SECOND BALCONY



POLL 1

(Fall 2017 Final Exam #11)

Find the area of the plane $x + 2y + 2z = 2$ that lies in the first octant.

a) 3

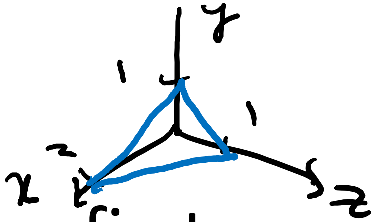
b) $\frac{\sqrt{5}}{2}$

c) $\frac{3}{2}$

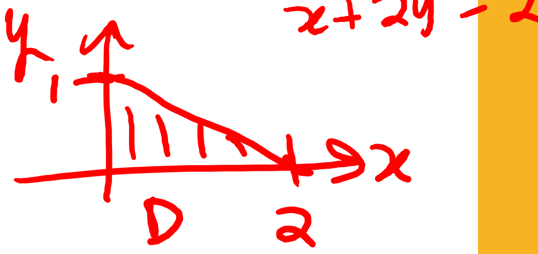
d) $\frac{3}{4}$

e) $\frac{\sqrt{5}}{4}$

$2z = 2 - x - 2y$



parameterize S



$\vec{r}(x,y) = \langle x, y, 1 - \frac{x}{2} - y \rangle$

$t_x = \langle 1, 0, -\frac{1}{2} \rangle$

$t_y = \langle 0, 1, -1 \rangle$

$|t_x \times t_y| = |\langle \frac{1}{2}, 1, 1 \rangle| = \sqrt{(\frac{1}{2})^2 + 1^2 + 1^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$\iint_S 1 \cdot dS = \iint_D \frac{3}{2}\, dA = \frac{3}{2}\,\text{area}(D)$

$= \frac{3}{2} \cdot \frac{1}{2} \cdot 2 \cdot 1 = \boxed{\frac{3}{2}}$

POLL 2

Which Theorem should you use for the problem below (if any)?

Let $\mathbf{F} = (y + z\cos(x))\,\mathbf{i} + (-x + z\sin(y))\,\mathbf{j} + (xye^z)\,\mathbf{k}$, compute

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS,$$

where S is the part of the graph of $z = f(x, y) = e^x\,(x^2 + y^2 - 36)$ below the xy-plane with downward pointing normal.

a) Fundamental Theorem of Line Integrals

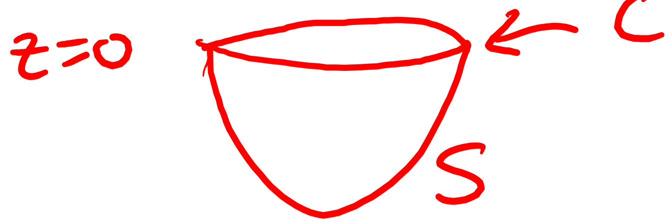
b) Green's Theorem

c) Stokes' Theorem

d) Divergence Theorem

e) None

← $\vec{F}$ is pretty complicated



open surface, difficult to parameterize

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dS = \oint_C \vec{F} \cdot d\vec{r}$$

@ $z = 0 = e^x (x^2 + y^2 - 36)$

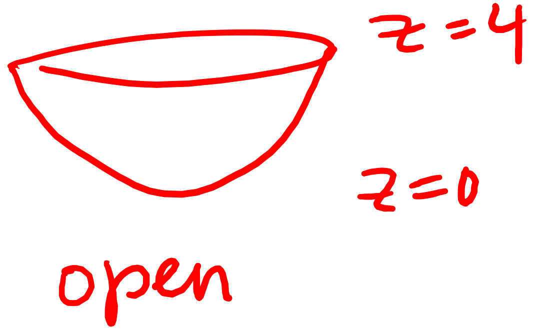
$x^2 + y^2 = 36$ ← circle of radius 6

POLL 3

Which Theorem should you use for the problem below (if any)?

PROBLEM 17: Let S be the part of paraboloid $z = x^2 + y^2$ which is below $z = 4$ with downward normal. Let $\mathbf{F}$ be the vector field $\langle y, x, z \rangle$. Compute the flux

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$



a) ~~Fundamental Theorem of Line Integrals~~

b) ~~Green's Theorem~~

c) ~~Stokes' Theorem~~

d) ~~Divergence Theorem~~

e) None

POLL 4

Which Theorem should you use for the problem below (if any)?

Compute $\oint_C y^2\,dx + x\,dy$, where the curve C is the boundary of the half-disk

$$R = \{(x,\, y) :\; x^2 + y^2 \leq 9 \text{ and } x \geq 0\}$$

with clockwise orientation.

a) Fundamental Theorem of Line Integrals
b) Green's Theorem
c) Stokes' Theorem
d) Divergence Theorem
e) None

POLL 5

Which Theorem should you use for the problem below (if any)?

PROBLEM 14: Let

$$\mathbf{F} = \langle y, x + e^z, ye^z + 1 \rangle$$

Calculate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the curve $r(t) = \langle t^2, t^3, \sin(t\pi) \rangle : 0 \leq t \leq 2$

a) Fundamental Theorem of Line Integrals
b) Green's Theorem
c) Stokes' Theorem
d) Divergence Theorem
e) None

POLL 6

Which Theorem should you use for the problem below (if any)?

Compute

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

the net outward flux of the vector field $\mathbf{F} = \langle x + y,\ y - z,\ xy + z \rangle$ across the surface S, which is the boundary of the solid bounded by $z = 0$, $y = 0$, $y + z = 2$, and $z = 1 - x^2$.

a) Fundamental Theorem of Line Integrals
b) Green's Theorem
c) Stokes' Theorem
d) Divergence Theorem
e) None

POLL 7

Which Theorem should you use for the problem below (if any)?

Let S be an open surface in $\mathbb{R}^3$ with boundary curve C where C is a circle in the $xy-$plane with radius 1 and center $(0,0,0)$ and is oriented counterclockwise when viewed from above. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x,y,z)$ is such that $\text{curl}(\vec{F}) = \langle \frac{1}{2}, -\frac{3}{2}, \frac{1}{4} \rangle$.

a) Fundamental Theorem of Line Integrals
b) Green’s Theorem
c) Stokes’ Theorem
d) Divergence Theorem
e) None

POLL 8

Which Theorem should you use for the problem below (if any)?

Let S be the part of the plane $y + z = 10$ that lies inside the cylinder $x^2 + y^2 = 1$. Compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ for $\mathbf{F}(x, y, z) = \langle x, 1 - y + e^z, y - e^z \rangle$ with S oriented by the upward normal.

a) Fundamental Theorem of Line Integrals
b) Green's Theorem
c) Stokes' Theorem
d) Divergence Theorem
e) None