Final Exam Review 2 - 12/6/2023 - 3:30pm

Scalar Integrals

Line Int.

$$\int_C f(x,y,z)\,ds$$

parameterize C
$\vec{r}(t)$ for $a \le t \le b$

$$= \int_a^b f(\vec{r}(t))\,|\vec{r}\,'(t)|\,dt$$

If $f \equiv 1$
$\to$ arclength

$$L = \int_C 1 \cdot ds = \int_a^b |\vec{r}\,'(t)|\,dt$$

Surface Int

$$\iint_S f(x,y,z)\,dS$$

parameterize S
$\vec{r}(u,v)$
where $(u,v) \in D$

$$= \iint_D f(\vec{r}(u,v))\,|\vec{t}_u \times \vec{t}_v|\,dA$$

If $f \equiv 1 \to$ surface area

NO Thms for scalar integrals
similar to Fubini's Thm
rewrite integrals as interated Calc I integrals

★ Vector Integrals & Theorems ★

$$\oint_C \vec{F} \cdot d\vec{r}$$

Stokes' Thm
if $\vec{G} = \vec{\nabla} \times \vec{F}$
and S open with boundary C

$$\iint_S \vec{G} \cdot \vec{n}\,dS$$

If $\vec{F} = \nabla\phi$
Fundamental Thm of Line Integrals

$$\phi(B) - \phi(A)$$

Divergence Thm
If $\vec{\nabla} \cdot \vec{G} = h(x,y,z)$

$$\iiint_E h(x,y,z)\,dV$$

Green's Thm (Circulation Form) $\vec{F} = \langle P, Q \rangle$ is 2D

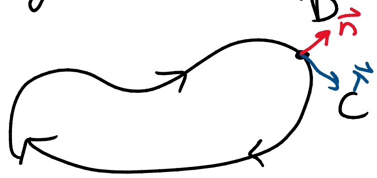
$$\oint_C \vec{F} \cdot \vec{T}\,ds = \oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$\leftarrow$ curl

$$\oint_C \vec{F}\cdot\vec{T}\,ds = \oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$ ← curl

Green's Thm (Flux Form)

$$\oint_C \vec{F}\cdot\vec{n}\,ds = \oint_C -P\,dy + Q\,dx = \iint_D \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dA$$ ← divergence



Given a surface integral

$$\iint_S \vec{G}\cdot\vec{n}\,dS$$

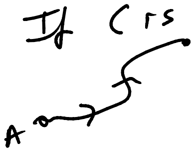
which Thm to use?

- If $\vec{G} = \vec{\nabla}\times\vec{F}$ and S is open → Stokes' Thm $= \oint_C \vec{F}\cdot d\vec{r}$
- If S is a closed surface → Divergence Thm $= \iiint_E \vec{\nabla}\cdot\vec{G}\,dV$
- If S is "simple" (plane, elliptic parabola) etc... → just compute $\iint_D G(\vec{F}(u,v))\cdot(\vec{r}_u\times\vec{r}_v)\,dA$

Given a line Integral

$$\int_C \vec{F}\cdot d\vec{r}$$

If C is open

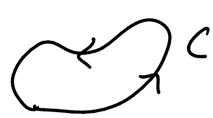


Check if $\vec{F}$ is conservative $\vec{F} = \vec{\nabla}\phi$

- yes → Fundamental Thm
- NO → just compute

if not complicated

If C is closed and complicated



Is $\vec{F}$ 2D or 3D?

- 2D → Green's
- 3D → Stokes' Thm

Fundamental Thm of Line Int $\phi(B)-\phi(A)$	just compute	2D Green's Thm	Stokes' Thm

	Curves	Surfaces
Open	line (A to B), Helix, parabola	elliptic parabola $z=x^2+y^2$; hemisphere $\rho=a$ for $z\geq 0$; cone $0\leq z\leq b$; plane; plane inside cylinder
Closed	circle, square, triangle, piecewise funs	sphere solid; box; open surfaces + the plane at the opening (solid cone); solid cylinder with $z=0$ and $z=b$