FINAL EXAM REVIEW 3

MA 26100·FALL 2023

DR. HOOD

WARM UP

Which Theorem should you use for the problem below (if any)?

Compute

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$$

the net outward flux of the vector field $\mathbf{F} = \langle x + y,\, y - z,\, xy + z \rangle$ across the surface S, which is the boundary of the solid bounded by $z = 0$, $y = 0$, $y + z = 1$ and $z = 1 - x^2$.

closed

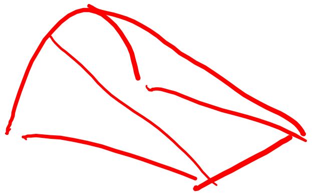
a) Fundamental Theorem of Line Integrals
b) Green's Theorem
c) Stokes' Theorem
d) Divergence Theorem
e) None

FINAL REVIEW 3

WARM UP

FINAL REVIEW 2

Compute



$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$$

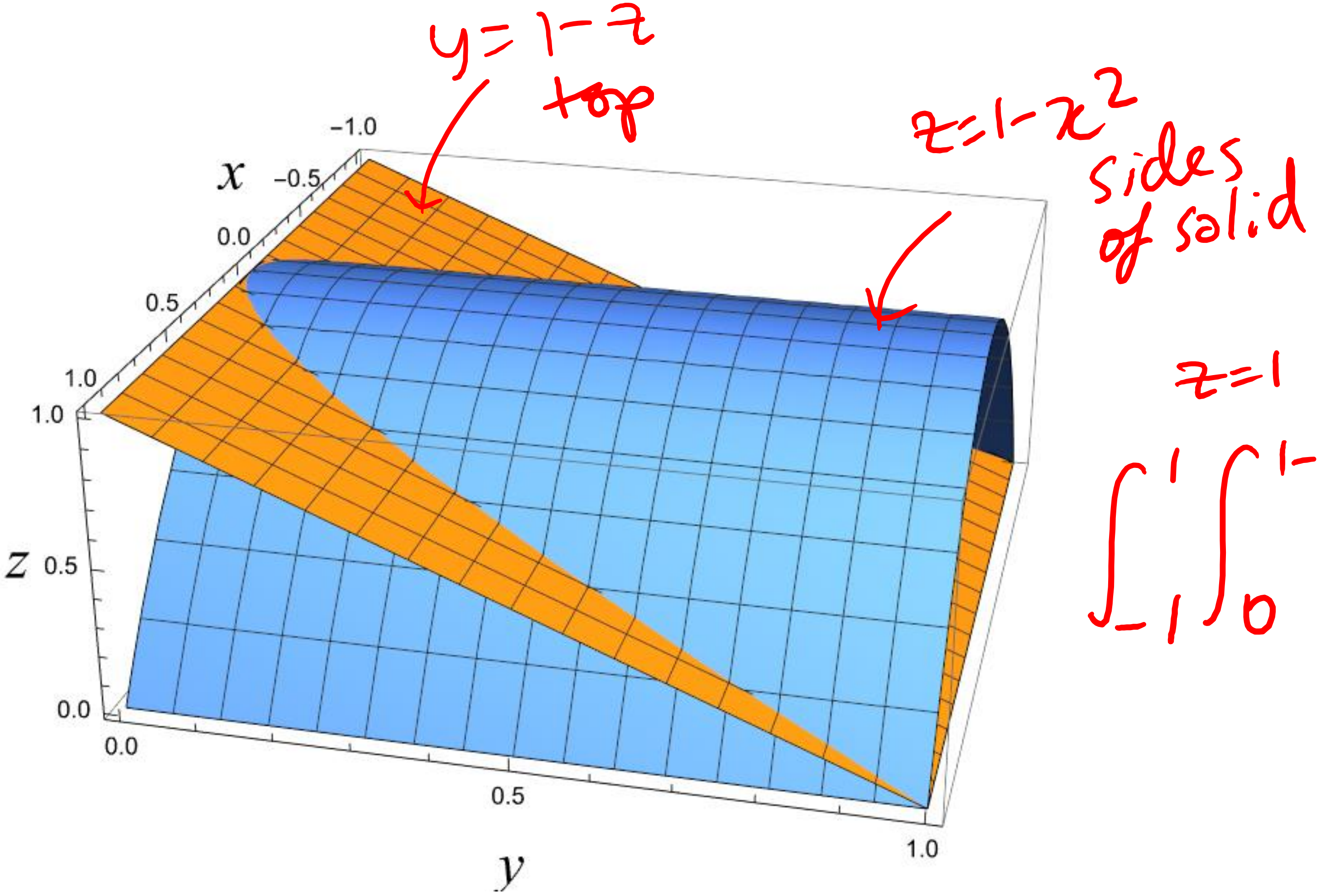
the net outward flux of the vector field $\mathbf{F} = \langle x+y,\ y-z,\ xy+z\rangle$ across the surface S, which is the boundary of the solid bounded by $z=0$, $y=0$, $y+z=2$, and $z=1-x^2$.

$-1 \le x \le 1$

$0 \le z \le 1-x^2$

$0 \le y \le 1-z$

1

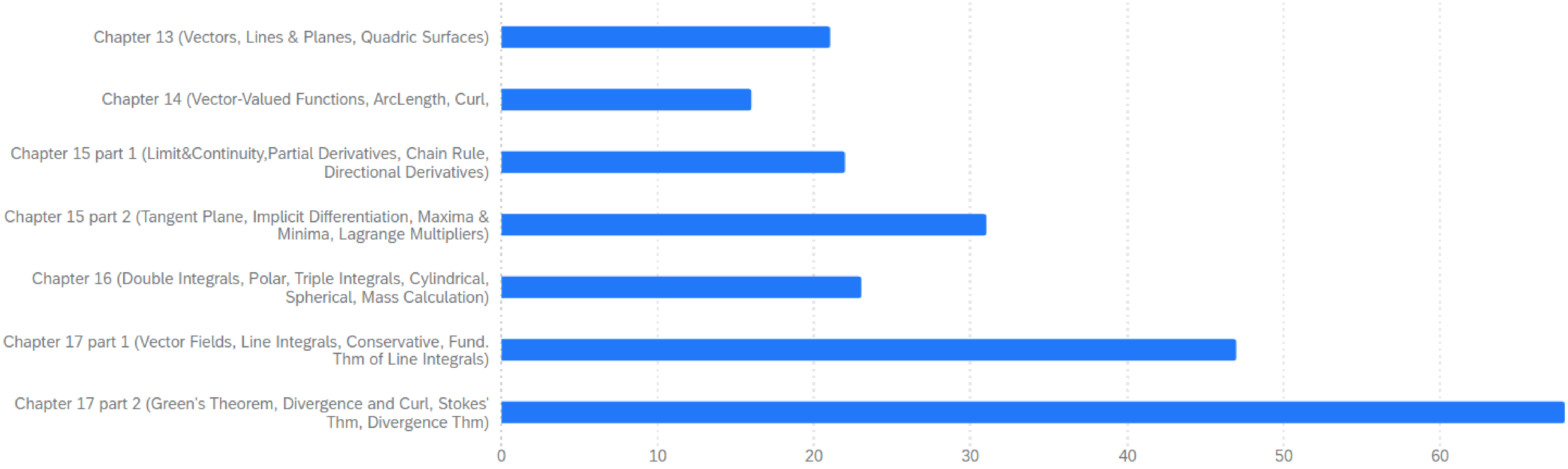


$z=1$

$$\int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{1-z} 3\,dy\,dz\,dx = \dots = \frac{12}{5}$$

SURVEY RESULTS

Which topics would you most like to cover in the review class on Friday? (Check your top 3) 87



ANNOUNCEMENTS

- **Final Exam is Mon Dec 11 at 8:00am- 10:00am**
  - Note: in the morning!
- **Please fill out course evaluations:** https://purdue.evaluationkit.com/MyEval/Login.aspx
- **New seating arrangements for the Final Exam:**
  - 23 TAs in ELLT
  - 1 TA in RHPH (Howen Chuah)

CLOSED VS OPEN SURFACES

- Open

one equation

17. If S is the portion of the cone $z = (x^2 + y^2)^{1/2}$ with $0 \leq z \leq 1$, then $\int\int_S z\, dS$ is

19. Let S be the part of the circular paraboloid $z = x^2 + y^2$ below the plane $z = 4$ with upward orientation. Let $\mathbf{F}(x, y, z) = xz\vec{\mathbf{j}} + yz\vec{\mathbf{k}}$. Compute $\iint \text{curl}\, \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, dS$. *Hint: You*

- Closed

19. Evaluate the outward flux $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F}(x, y, z) = 3x\vec{i} + 4x^2\vec{j} + y^2\vec{k}$ and S is the closed surface (sides, top, and bottom) of the cylinder $x^2 + y^2 = 1$ between the planes $z = 0$ and $z = 1$.

20. Find the outward flux of the vector field $\vec{F} = \langle \sin y, xz, 3z \rangle$ across the boundary of the solid space between two spheres of radii 1 and 2, both centered at the origin.

(Fall 2017 Final Exam #19)

19. Let S be the upper hemisphere of $x^2 + y^2 + z^2 = 4$ with normal vector pointing toward the origin, and $\mathbf{F} = z\dfrac{\mathbf{x}}{|\mathbf{x}|}$ where $\mathbf{x}$ denotes the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = \langle x, y, z\rangle$.

Compute $\displaystyle\iint_S \mathbf{F} \cdot d\mathbf{S}$.

A. -8π

B. 8π

C. -4π

D. 4π

E. 0

(Practice Final Exam Ver A #20)

Problem 20 (5 points)

Use Stokes' Theorem to evaluate the integral $\int_C y\,dx + z\,dy + x\,dz$, where C is the intersection of the surfaces $x^2 + y^2 = 1$ and $x + y + z = 5$. C is oriented counterclockwise when viewed from above.

A. -6π

B. -3π

C. -9π

D. -8π

E. $-\pi$

(Fall 2019 Final Exam #18)

18. Consider $\mathbf{F} = \dfrac{\mathbf{r}}{|\mathbf{r}|^3}$, where $\mathbf{r} = \langle x, y, z \rangle$ and $|\mathbf{r}| = (x^2 + y^2 + z^2)^{1/2}$. Which one of the following is true

(i) $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path.

(ii) $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = 0$ for any closed surface S that encloses the origin.

(iii) $div(\mathbf{F}) = 0$.

A. None of the above.

B. Only (i) and (ii).

C. Only (i) and (iii)

D. Only (ii) and (iii).

E. All of the above.

(Practice Final Exam Ver C #9)

Problem 9

Consider the function

$$f(x,y) = \frac{1}{4}x^4 + xy + \frac{1}{4}y^4 \text{ on } R^2$$

Then the function

A. has one saddle point and two local minima.

B. has one local maximum and two local minima.

C. has an absolute maximum and absolute minimum.

D. has 4 critical points.

E. is always positive and hence has absolute minimum of 0.

(Spring 2016 Final Exam #11)

11. Find the formulas for a and b that convert the triple integral from rectangular coordinates to spherical coordinates:

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{x^2+y^2}} x\,dz\,dy\,dx = \int_{0}^{\pi}\int_{a}^{\frac{\pi}{2}}\int_{0}^{2\csc\phi} b\,d\rho\,d\phi\,d\theta$$

A. $a = \frac{\pi}{4}, b = \rho^3 \sin^2\phi\cos\theta$

B. $a = 0, b = \rho^3 \sin^2\phi\cos\theta$

C. $a = 0,\ b = \rho\sin\phi\cos\theta$

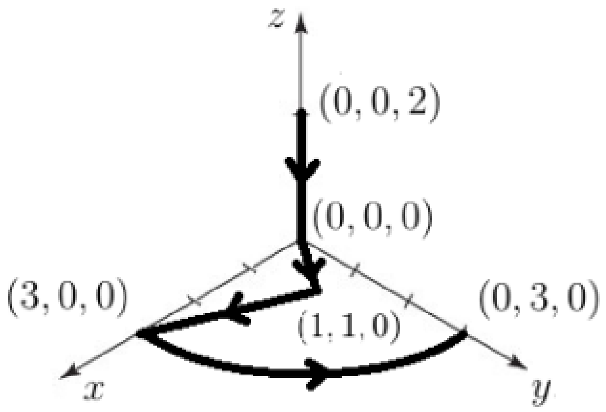
D. $a = \frac{\pi}{4}, b = \rho^3 \sin\phi$

E. $a = 0, b = \rho^2 \sin\phi\cos\theta$

(Fall 2017 Final Exam #12)

12. The oriented curve C consists of the line segment from $(0,0,2)$ to $(0,0,0)$, followed by the line segment from $(0,0,0)$ to $(1,1,0)$, followed by the line segment from $(1,1,0)$ to $(3,0,0)$, followed by the circular arc from $(3,0,0)$ to $(0,3,0)$, as shown in the figure below.

Find the value of $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ with vector field $\mathbf{F}(x,y,z) = ye^x\mathbf{i} + e^x\mathbf{j} + 2z\mathbf{k}$.



A. -2

B. -1

C. 0

D. 1

E. 2