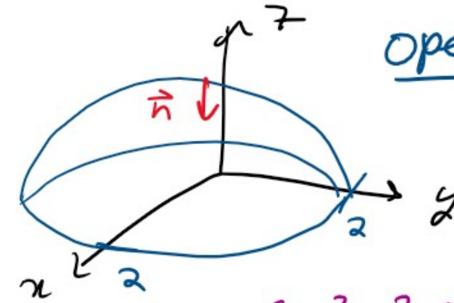
F2017 FE #19

$x^2+y^2+z^2=4$ normal pts toward origin

$\vec{F} = z\dfrac{\vec{x}}{|\vec{x}|}$ $\vec{x} = \langle x, y, z\rangle$

Compute $\iint_S \vec{F}\cdot d\vec{S}$



Open

Open $\rightarrow$ NOT Div Thm

No curl $\rightarrow$ NOT Stokes' Thm

Compute directly $\rightarrow$ parameterize S

Implicitly $x^2+y^2+z^2-4=0=G$

$$\frac{\partial z}{\partial x} = -\frac{G_x}{G_z} = \frac{-2x}{2z} = \frac{-x}{z}$$

$$\frac{\partial z}{\partial y} = -\frac{G_y}{G_z} = -\frac{2y}{2z} = \frac{-y}{z}$$

$$\vec{r}(x,y) = \langle x, y, z(x,y)\rangle$$

$$\vec{t}_x = \left\langle 1, 0, \frac{\partial z}{\partial x}\right\rangle = \left\langle 1, 0, \frac{-x}{z}\right\rangle$$

$$t_y = \left\langle 0, 1, \frac{\partial z}{\partial y}\right\rangle = \left\langle 0, 1, \frac{-y}{z}\right\rangle$$

$$t_x \times t_y = \left\langle \frac{x}{z}, \frac{y}{z}, 1\right\rangle \qquad \text{take } \vec{n} = \left\langle \frac{-x}{z}, \frac{-y}{z}, -1\right\rangle$$

$|\vec{x}| = \sqrt{x^2+y^2+z^2} = \sqrt{4} = 2$

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iint_D z\left\langle \frac{x}{\cancel{|\vec{x}|}_{2}}, \frac{y}{\cancel{|\vec{x}|}_{2}}, \frac{z}{\cancel{|\vec{x}|}_{2}}\right\rangle \cdot \left\langle -\frac{x}{z}, \frac{-y}{z}, -1\right\rangle dA$$

$$= \frac{1}{2}\iint_D \cancel{z}\left(\frac{-x^2}{\cancel{z}} - \frac{y^2}{\cancel{z}} - \frac{z^2}{\cancel{z}}\right) dA = \frac{1}{2}\iint -\underbrace{(x^2+y^2+z^2)}_{2^2} dA$$

$$= \frac{-4}{2}\iint_D dA$$

shadow of hemisphere is xy plane disk of radius 2

$$= -2\,\text{area(disk)} = -2\pi(2)^2 = \boxed{-8\pi}$$

Can also use spherical coordinates $\boxed{\rho = 2}$

$$\vec{r}(\theta,\phi) = \langle 2\sin\phi\cos\theta, 2\sin\phi\sin\theta, 2\cos\phi\rangle$$

$$t_\phi = \langle 2\cos\phi\cos\theta, 2\cos\phi\sin\theta, -2\sin\phi\rangle$$

$$\langle -2\sin\phi\sin\theta, 2\sin\phi\cos\theta, 0\rangle$$

$t_\phi = \langle 2\cos\phi\cos\theta,\ 2\cos\phi\sin\theta, \ldots$

$t_\theta = \langle -2\sin\phi\sin\theta,\ 2\sin\phi\cos\theta,\ 0 \rangle$

$t_\phi \times t_\theta = \ldots = 2\sin\phi \langle \underbrace{2\sin\phi\cos\theta}_{x}, \underbrace{2\sin\phi\sin\theta}_{y}, \underbrace{2\cos\phi}_{z} \rangle$

$= 2\sin\phi \langle x, y, z \rangle$

$= 2^2 \sin\phi \langle \frac{x}{2}, \frac{y}{2}, \frac{z}{2} \rangle = \boxed{2^2 \sin\phi \left(\frac{\vec{r}}{|\vec{r}|}\right)}$

$$\iint_S \vec{F}\cdot d\vec{S} = -\int_0^{2\pi}\int_0^{\frac{\pi}{2}} z\langle \tfrac{x}{2}, \tfrac{y}{2}, \tfrac{z}{2} \rangle \cdot 2^2 \sin\phi \langle \tfrac{x}{2}, \tfrac{y}{2}, \tfrac{z}{2} \rangle \, d\phi\, d\theta$$

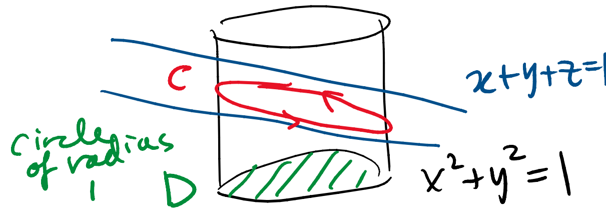
$$= \frac{2^2 \cdot \cancel{2^2}}{\cancel{2^2}} \int_0^{2\pi}\int_0^{\frac{\pi}{2}} 2\cdot\cos\phi \sin\phi \, d\phi\, d\theta \overset{\ldots}{=} -8\pi$$

Practice Final Ver A #20

Stokes' Thm

$\int_C y\,dx + z\,dy + x\,dz$ $\qquad$ C intersection of $x^2+y^2=1$ and $x+y+z=1$

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\, dS$$



surface — plane

$\vec{r}(x,y) = \langle x, y, 1-x-y \rangle$

$t_x = \langle 1, 0, -1 \rangle$

$t_y = \langle 0, 1, -1 \rangle$

$t_x \times t_y = \langle 1, 1, 1 \rangle$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix} = \langle -1, -1, -1 \rangle$$

$$= \iint_D \langle -1,-1,-1 \rangle \cdot \langle 1,1,1 \rangle \, dA = -3\iint_D dA$$

$$= -3\ \text{area}(D) = -3\pi(1)^2 = \boxed{-3\pi}$$

$= -3 \text{ area}(D) = -3\pi(1)^2 \ldots$

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iint_D \vec{F}(r(u,v))\cdot\frac{(\vec{t_u}\times\vec{t_v})}{\cancel{|t_u\times t_v|}}\,\cancel{|t_u\times t_v|}\,dA$$

$$= -3\int_0^{2\pi}\int_0^1 r\,dr\,d\theta$$

$\vec{F} = \frac{\vec{r}}{|\vec{r}|^3}$ $\quad \vec{r} = \langle x, y, z\rangle$

(i) $\int_C \vec{F}\cdot d\vec{r}$ is independent of path $\quad \vec{F} = \vec{\nabla}\phi$

$\vec{F}$ is conservative :

$\vec{F} = \langle P, Q, R\rangle \quad \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}, \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y},$

$\frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}$

(iii) $\vec{\nabla}\cdot\vec{F} = 0$ if $(x,y,z) \neq (0,0,0)$

(ii) $\iint_S \vec{F}\cdot\vec{n}\,dS = 0$ for any closed surface encloses origin

$\neq \iiint_E \vec{\nabla}\cdot\vec{F}\,dV$ because $\vec{F}$ and $\vec{\nabla}\cdot\vec{F}$ is undefined at $(0,0,0)$

$S = S_a$ sphere radius a

$$\iint_{S_a} \frac{\vec{r}}{|\vec{r}|^3}\cdot\frac{\vec{r}}{|a|}\cdot a^2\sin\phi\,d\phi\,d\theta$$

$$= \frac{\cancel{a^2}\cdot\cancel{a^2}}{\cancel{a^4}}\int_0^{2\pi}\int_0^{\pi}\sin\phi\,d\phi\,d\theta = \boxed{4\pi} \quad \boxed{\text{FALSE}}$$

Maxima & Minima

$f = \frac{x^4}{4} + xy + \frac{y^4}{4}$

critical points:

$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$

$x^3 + y = 0 \qquad x + y^3 = 0$

$$f = \frac{x^4}{4} + xy + \frac{y^4}{4}$$

∂x

$x^3 + y = 0$

$y = -x^3$

when $x = 0 \quad y = 0$

$x = 1 \quad y = -1$

$x = -1 \quad y = +1$

$x + y^3 = 0$

$x + (-x^3)^3 = 0$

$x - x^9 = 0$

$x(1 - x^8) = 0$

$x = 0 \qquad x = \pm 1$

Discriminant $D = f_{xx} f_{yy} - (f_{xy})^2$

$f_{xx} = 3x^2$

$f_{xy} = 1$

$f_{yy} = 3y^2$

@ $(0,0)$ $\quad D = 0 \cdot 0 - (1)^2 = -1$ saddle

@ $(1,-1)$ $\quad D = 3 \cdot 3 - (1)^2 = 8$ ++ min

$f_{xx} = 3$

@ $(-1,1)$ $\quad D = 3 \cdot 3 - (1)^2 = 8$ ++ min

$f_{xx} = 3$

$D < 0 \longrightarrow$ saddle

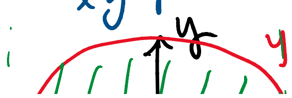
$D > 0 \quad f_{xx} > 0$ ++ min

$\qquad\quad f_{xx} < 0$ −− max

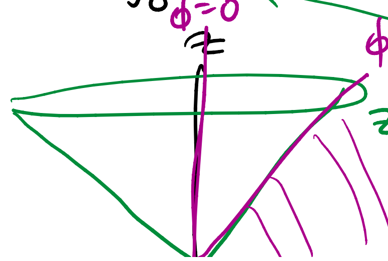
$D = 0 \longrightarrow$ Inconclusive

$\boxed{\text{Sp 16 FE \#11}}$

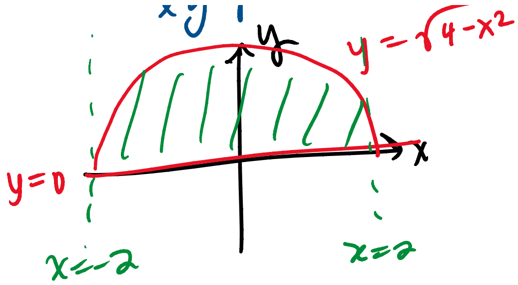
$$\int_{-2}^{2} \underbrace{\int_{0 = y}^{\sqrt{4-x^2}}}_{xy \text{ plane}} \int_0^{\sqrt{x^2+y^2} = z} x \, dz\,dy\,dx = \int_0^{\pi} \int_{a}^{\frac{\pi}{2}} \int_0^{2\csc\phi} b \, d\rho\, d\phi\, d\theta$$



$0 \le \theta \le \pi$



$z = r \Rightarrow \phi = \frac{\pi}{4}$



$0 \le \theta \le \pi$