## Final Exam Study Guide

MA 261 • Fall 2023
The Final Exam will be 20 multiple choice questions. It is comprehensive, covering Lessons $1-37$.
Past Exam Archive: https://www.math.purdue.edu/academic/courses/oldexams.php?course=MA26100
A breakdown of the learning objectives, with practice problems from the textbook and past exam problems is included in the table below.

| \# | Lesson: | Sec: | Quiz: | You should be able to: | You should know: | Practice Problems: | Past Exam Problems: |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Review of Vectors | $\begin{aligned} & 13.1 \\ & 13.2 \\ & 13.3 \\ & 13.4 \end{aligned}$ | 1 | - Compute vector operations <br> - Find magnitude of a vector <br> - Find a position vector <br> - Find the equation of a sphere, ball, or circle <br> - Find equations of simple planes <br> - Compute dot products <br> - Find angles between vectors <br> - Calculate orthogonal projections <br> - Compute cross products <br> - Find areas of parallelograms and triangles <br> - Find orthogonal vectors | Vector, scalar, magnitude, zero vector, position vector, unit vector, parallel, sphere, ball, circle, plane, dot product, orthogonal, orthogonal projection, cross product, determinant, coordinate unit vectors (i, $\mathbf{j}, \mathbf{k}$ ) | $\begin{aligned} & \text { 13.1: \# 21, 23, 29, } \\ & 43,45,47 \\ & \text { 13.2: \# 31, } 37,41 \text {, } \\ & 45,71,73 \\ & \text { 13.3: \# 25, 27, 35, } \\ & 37,39,41,43,47 \text {, } \\ & 53,55,63,65 \\ & \text { 13.4: \# 13, 21, 25, } \\ & 27,29,33,43,45, \\ & 49 \end{aligned}$ | Few exam questions test these concepts directly. However, you will need these concepts to complete the more difficult questions from the past exams. <br> S18E1\#4 <br> S18FE\#1 <br> S16E1\#1 |
| 2 | Lines \& Planes in Space | 13.5 | 1 | - Find equations of lines and line segments <br> - Find equations of planes <br> - Determine whether planes are parallel, intersecting, or identical - Find intersections between lines and/or planes | Parallel, intersecting, skew, orthogonal planes | $\begin{aligned} & \text { 13.5: \# 21, 23, 25, } \\ & 49,51,55,57,65, \\ & 67,69,73,75,77 \text {, } \\ & 79,81,90 \end{aligned}$ | $\begin{aligned} & \text { S19E1\#1 } \\ & \text { S19E1\#2 } \\ & \text { S19FE\#1 } \\ & \text { F19E1\#1 } \\ & \text { F19FE\#1 } \\ & \text { F18E1\#1 } \\ & \text { F18FE\#1 } \end{aligned}$ |
| $\begin{aligned} & \hline 3 \\ & 4 \end{aligned}$ | Quadratic Surfaces | 13.6 | 2 | - Sketch graphs of cylinders and quadratic surfaces <br> - Identify surfaces from equations | Trace, elliptic paraboloid, ellipsoid, cylinder, elliptic cone, hyperboloid of one sheet, hyperboloid of two sheets, hyperbolic paraboloid | $\begin{aligned} & \text { 13.6: \# 15, 17, 19, } \\ & 21,23,25,27,31, \\ & 35,37,39,43,45, \\ & 47,49,51,55,57, \\ & 59 \end{aligned}$ | $\begin{aligned} & \hline \text { S19FE\#2 } \\ & \text { F19E1\#2 } \\ & \text { S18E1\#1 } \\ & \text { F18E1\#3 } \\ & \text { F18FE\#2 } \end{aligned}$ |


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| 5 | Vector-Valued Functions | 14.1 | 2 | - Graph curves described by vectorvalued functions <br> - Find domains of vector-valued functions <br> - Find the intersection of planes and curves defined by vector-valued functions | Vector-valued function, domain, limit of a vector-valued function | $\begin{aligned} & \text { 14.1: \# 11, 12, 15, } \\ & \text { 17, 19, 21, 39, 41, } \\ & 43,45,47,49,53, \\ & 55 \end{aligned}$ | S22E1\#4 <br> S19E1\#3 <br> F19FE\#2 <br> F18E1\#2 <br> F16E1\#4 <br> S14E1\#9 |
| 6 | Calculus of Vector-Valued Functions, Motion in Space | $\begin{aligned} & 14.2 \\ & 14.3 \end{aligned}$ | 3 | - Find first derivatives of vectorvalued functions <br> - Find tangent vectors and tangent lines for vector-valued functions - Evaluate definite integrals of vector-valued functions <br> - Find velocity, speed, and acceleration of objects | Tangent vector, unit tangent vector, tangent line, derivative rules | $\begin{aligned} & \text { 14.2: \# 11, 13, 15, } \\ & \text { 17, 19, 21, 25, 27, } \\ & \text { 29, 35, 37, 39, 47, } \\ & 71,73,75,77,79, \\ & 81,83 \\ & \text { 14.3: \# 47, } 49 \end{aligned}$ | S18E1\#2 <br> S18E1\#3 <br> S17E1\#3 <br> S16E1\#5 |
| 7 | Motion in Space | 14.3 | 3 | - Compare trajectories of objects <br> - Solve applications involving 2d and 3d motion | Velocity, acceleration, trajectories | $\begin{aligned} & \text { 14.3: \# 15, 17, 19, } \\ & 29,31,47,49,57 \end{aligned}$ | S19E1\#6 <br> S19FE\#20 <br> F19E1\#3 <br> F19E1\#6 <br> S18FE\#3 <br> F18E1\#5 <br> F18E1\#6 |
| 8 | Length of Curves, Curvature | $\begin{aligned} & \hline 14.4 \\ & 14.5 \end{aligned}$ | 3 | - Find arc lengths of vector-valued functions <br> - Parameterize curves by arc length <br> - Find unit tangent vectors and curvatures <br> - Use velocity to find curvature | Arc length, curvature | $\begin{aligned} & \text { 14.4: \# 9, 11, 13, } \\ & \text { 15, 17, 19, 23, 25, } \\ & 33,35,37 \\ & \text { 14.5: \# 11, 13, 15, } \\ & 17,21,23,25 \end{aligned}$ | Arc Length <br> S19E1\#5 <br> F19E1\#5 <br> F19FE\#3 <br> S18FE\#2 <br> Curvature <br> S19E1\#4 <br> F19E1\#4 <br> F18E1\#4 |
| 9 | Functions of Several Variables | 15.1 | 4 | - Find domains of functions <br> - Graph surfaces <br> - Graph level curves of functions | Function of several variables, level curves | $\begin{aligned} & \text { 15.1: \# 15, 19, 21, } \\ & 23,27,29,31,33, \\ & 35,37,39,41,57 \end{aligned}$ | $\begin{aligned} & \hline \text { S19E1\#7 } \\ & \text { S18E1\#5 } \\ & \text { F18E1\#7 } \end{aligned}$ |


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| 10 | Limits and Continuity | 15.2 | 4 | - Evaluate limits of functions <br> - Evaluate limits at boundary points <br> - Determine where functions are continuous | Limit laws, boundary point, interior point, two-path test, continuity | $\begin{aligned} & \text { 15.2: \# 17, 19, 21, } \\ & 23,25,27,29,31, \\ & 33,39,41,49,53, \\ & 61,63,65,67,69, \\ & 71,77 \end{aligned}$ | F19E1\#7 <br> F18E1\#8 <br> S17E1\#6 |
| 11 | Partial Derivatives | 15.3 | 4 | - Find first partial derivatives <br> - Find second partial derivatives | Partial derivative, differentiable, | $\begin{aligned} & \text { 15.3: \# 17, 19, 21, } \\ & 27,33,39,41,43, \\ & 45,47,55,57,79 \end{aligned}$ | S19E1\#8 <br> S19FE\#7 <br> F19E1\#8 <br> F19FE\#6 <br> F18FE\#5 |
| 12 | The Chain Rule | 15.4 | Practice | - Use the chain rule to find derivatives <br> - Differentiate implicitly <br> - Evaluate partial derivatives at specified points | Chain rule, implicit differentiation | $\begin{aligned} & \text { 15.4: \# } 15,17,19, \\ & 21 \quad 23 \quad 35 \quad 3739, \end{aligned}$ | S19E1\#9 <br> F19FE\#7 <br> S18FE\#5 <br> F18E1\#10 |
| 13 | Directional Derivatives and the Gradient | 15.5 | Practice | - Compute gradients and/or directional derivatives <br> - Find directions or paths of change <br> - Compute slopes of lines tangent to level curves | Gradient, directional derivative, directions of change, level curves, steepest descent | $\begin{aligned} & \text { 15.5: \#17, 19, 25, } \\ & 27,29,31,33,35, \\ & 61,63,65,67 \end{aligned}$ | $\begin{aligned} & \text { S19E1\#10 } \\ & \text { S19FE\#8 } \\ & \text { F19E1\#9 } \\ & \text { F19FE\#4 } \\ & \text { S18FE\#7 } \\ & \text { F18E1\#11 } \\ & \text { F18FE\#4 } \end{aligned}$ |
| 14 | Tangent Plane and Linear Approximation | 15.6 | 5 | - Find equations of planes tangent to surfaces <br> - Find linear approximations <br> - Use differentials to approximate changes in functions | Tangent plane, differential, linear approximation | $\begin{aligned} & \text { 15.6: \#13, 15, 17, } \\ & 19,21,25,29,33, \\ & 35,37,53 \end{aligned}$ | S19E1\#11 <br> S19FE\#5 <br> F19FE\#5 <br> S18FE\#6 <br> F18E1\#9 <br> F18FE\#3 |
| $\begin{aligned} & 15 \\ & 16 \end{aligned}$ | Maximum and Minimum Problems | 15.7 | 5 | - Find and analyze critical points for functions <br> - Find local and absolute extrema for functions | Local extrema, critical point, saddle point, second derivative test, absolute extrema | $\begin{aligned} & \text { 15.7: \#9, 11, 25, } \\ & 27,29,31,33,35, \\ & 37,39,41,47,49, \\ & 51,53,55,63,67 \end{aligned}$ | S19E1\#12 <br> S19FE\#9 <br> F19E1\#10 <br> F19E1\#11 <br> F19FE\#8 <br> S18FE\#9 <br> F18FE\#7 <br> F18E1\#12 <br> F18E2\#1 |


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| 17 | Lagrange Multipliers | 15.8 | 6 | - Use Lagrange multipliers to find extreme values | Lagrange multiplier | $\begin{aligned} & \text { 15.8: \#7, 9, 11, 13, } \\ & \text { 15, 17, 21, 23, 25, } \\ & 3541 \end{aligned}$ | S19E2\#1 <br> F19E2\#1 <br> F18FE\#6 <br> F18E2\#2 |
| 18 | Double Integrals in Rectangular Regions | 16.1 | 6 | - Evaluate iterated integrals <br> - Evaluate double integrals over rectangular regions <br> - Compute average values of functions over plane regions | Double integral, average value | $\begin{aligned} & \text { 16.1: \#5, 27, 29, } \\ & 31,33,35,45,47 \end{aligned}$ | $\begin{aligned} & \text { F19E2\#2 } \\ & \text { S18E2\#2 } \\ & \text { F18E2\#3 } \end{aligned}$ |
| 19 | Double Integrals over General Regions | 16.2 | 6 | - Evaluate double integrals over general regions <br> - Change the order of integration | Order of integration | $\begin{aligned} & \text { 16.2: } \# 43,47,51, \\ & 55,57,59,61,63, \\ & 65,67 \end{aligned}$ | S19E2\#2 S19E2\#4 S19FE\#10 F19E2\#3 F19FE\#9 F18E2\#4 |
| 20 | Double Integrals in Polar Coordinates | 16.3 | 6 | - Find volumes of solids using polar coordinates <br> - Evaluate double integrals using polar coordinates | Polar coordinates | $\begin{aligned} & \text { 16.3: \#15, 17, 21, } \\ & 23,25,27,29,31, \\ & 33,41,51,55 \end{aligned}$ | S19E2\#3 <br> S19FE\#11 <br> F18FE\#8 <br> F18E2\#5 |
| 21 | Triple Integrals | 16.4 | 7 | - Find volumes of solids using triple integrals <br> - Evaluate triple integrals <br> - Change the order of integration | Triple integrals | 16.4: \#15, 17, 19, 21, 25, 31, 37, 39, $41,45,47,49,55$ | S19E2\#5 S19FE\#12 F19E2\#4 F18FE\#9 F18E2\#7 |
| $\begin{aligned} & 22 \\ & 23 \end{aligned}$ | Triple Integrals in Cylindrical and Spherical Coordinates | 16.5 | 7 | - Evaluate triple integrals in cylindrical and spherical coordinates | Cylindrical coordinates, spherical coordinates | 16.5: \#15, 17, 19, 21, 25, 29, 31, 33, 41, 43, 45, 47, 49, $51,53,55$ | Cylindrical <br> S19E2\#6 <br> S19FE\#13 <br> F19E2\#5 <br> F19FE\#10 <br> S18FE\#8 <br> F18E2\#9 <br> F18E2\#8 <br> Spherical <br> S19E2\#7 <br> S19FE\#14 |


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|  |  |  |  |  |  |  | F19E2\#6 <br> F19E2\#7 <br> F18E2\#10 <br> F18FE\#10 <br> F18FE\#11 |
| 24 | Integrals for Mass Calculation | 16.6 | Practice | - Find centers of mass of 2D and 3D objects <br> - Calculate the mass of variable density solids | Center of mass, variable density | $\begin{aligned} & \text { 16.6: \#21, 23, 25, } \\ & 29,31,33,35,37, \\ & 39 \end{aligned}$ | $\begin{aligned} & \text { F19E2\#8 } \\ & \text { S18FE\#10 } \\ & \text { F18E2\#6 } \end{aligned}$ |
| 25 | Vector Fields | 17.1 | Practice | - Graph vector fields <br> - Find gradient fields for a given potential function | Vector field, radial vector field, potential function, equipotential curves, flow curves, streamlines. | $\begin{aligned} & \text { 17.1: \#11, } 13,15, \\ & 24,37,39,41,43, \\ & 47,53 \end{aligned}$ | S19E2\#8 F19E2\#9 F18E2\#11 F18E2\#12 |
| $\begin{aligned} & 26 \\ & 27 \end{aligned}$ | Line Integrals of Functions and Vector Fields | 17.2 | 8 | - Evaluate line integrals <br> - Find the work required to move an object on an oriented curve <br> - Find the circulation and flux of a vector field on a plane curve | Line integral, work, circulation, flux | $\begin{aligned} & \text { 17.2: \#17, 19, 21, } \\ & 23,25,27,29,31, \\ & 33,41,43,45,49, \\ & 51,53,57 \end{aligned}$ | Scalar: <br> S19E2\#10 <br> F19E2\#10 <br> F19E2\#11 <br> F19FE\#11 <br> Vector: <br> S19E2\#9 <br> S19FE\#15 <br> S18FE\#12 <br> F18FE\#12 |
| 28 | Conservative Vector Fields \& the Fundamental Theorem of Line Integrals | 17.3 | 8 | - Determine whether a vector field is conservative and find potential functions <br> - Evaluate line integrals <br> - Compute the work done in force fields | Conservative vector field, potential function, Fundamental Theorem for Line Integrals, independent of path | ```17.3: #17, 19, 23, 25, 35, 37, 39, 41, 43,53,55, 57, 59, 61``` | S19FE\#3 <br> F19FE\#12 <br> F18FE\#13 |
| 29 | Green's Theorem | 17.4 | 9 | - Use a line integral to determine the area of a region <br> - Use Green's theorem to evaluate line integrals <br> - Find the circulation and flux across the boundary of a region | Green's Theorem, two-dimensional curl, two-dimensional divergence, stream function, Laplace's equation | $\begin{aligned} & \text { 17.4: \#21, 27, 29, } \\ & 31,33,35,37,41, \\ & 49 \end{aligned}$ | S19FE\#4 <br> F19FE\#13 <br> F19FE\#14 <br> S18FE\#13 <br> F18FE\#16 <br> F18FE\#14 |


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| 30 | Divergence \& Curl | 17.5 | 9 | - Find the divergence of vector fields <br> - Find the curl of vector fields | Divergence, Curl, source-free, irrotational | $\begin{aligned} & \text { 17.5: \#13, 15, 17, } \\ & 21,25,27,29,31, \\ & 33,39,41 \end{aligned}$ | S19FE\#6 F19FE\#15 S18FE\#14 F18FE\#15 |
| $\begin{aligned} & 31 \\ & 32 \\ & 33 \end{aligned}$ | Surface Integrals | 17.6 | 10 | - Find a parametric description of a surface and describe surfaces parametrically <br> - Find the surface area using the parametric description of a surface - Evaluate surface integrals <br> - Evaluate flux integrals | Surface integral | $\begin{aligned} & \text { 17.6: \#19, 21, 23, } \\ & 25,27,29,31,33, \\ & 43,45,47,49,51 \end{aligned}$ | Scalar <br> S19E2\#4 <br> S19FE\#16 <br> S19FE\#17 <br> S18FE\#15 <br> S18FE\#16 <br> S18FE\#17 <br> F19FE\#16 <br> F18FE\#17 <br> Vector <br> S18FE\#18 <br> F19FE\#17 <br> F18FE\#18 |
| $\begin{aligned} & \hline 34 \\ & 35 \end{aligned}$ | Stokes' Theorem | 17.7 | Practice | - Use Stokes' Theorem to evaluate line integrals and surface integrals - Use Stokes' Theorem to find circulation | Stokes' Theorem | $\begin{aligned} & \text { 17.7: } \# 11,13,15, \\ & \text { 17, 19, 21, } 23,29, \\ & 41 \end{aligned}$ | S19FE\#18 <br> F19FE\#19 <br> S18FE\#19 <br> F18FE\#19 |
| $\begin{aligned} & 36 \\ & 37 \end{aligned}$ | The Divergence Theorem | 17.8 | Practice | - Use the Divergence Theorem to compute net outward flux | Divergence Theorem | $\begin{aligned} & \text { 17.8: \#13, 15, 17, } \\ & \text { 19, 21, 23, 25, 27, } \\ & 29,31,33,35, \end{aligned}$ | S19FE\#19 <br> F19FE\#18 <br> F19FE\#20 <br> S18FE\#20 <br> F18FE\#20 |

*Answers to the practice problems are included in the textbook starting on page A-61.

