



LESSON 10

MA 26100-FALL 2023

DR. HOOD

LESSON 10 – WARM UP

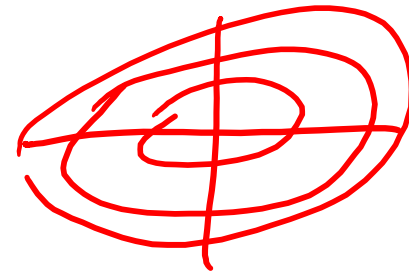
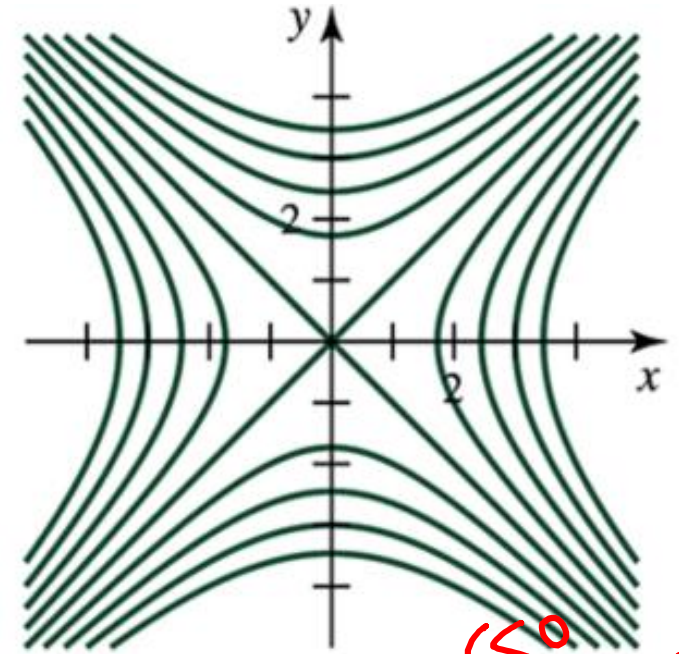
(Spring 22 Exam 1 #5) Suppose $z = f(x, y)$ has level curves shown here. The surface formed by the graph of f could be which of the following?

a) Hyperboloid of 2 sheets

b) Elliptic paraboloid

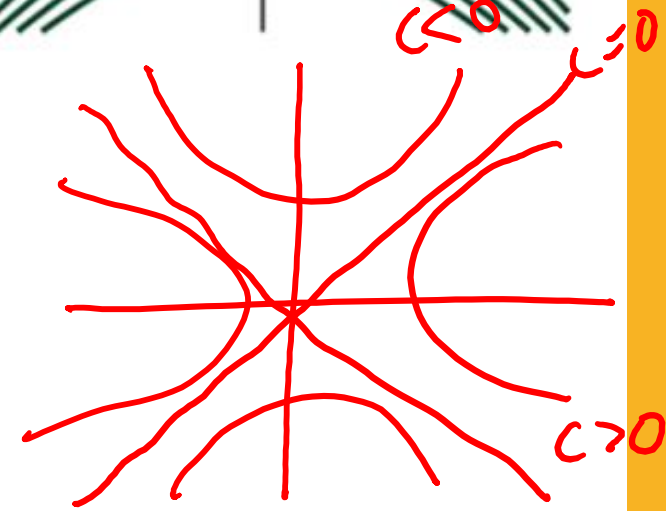
~~c) Elliptic cone~~

d) Hyperbolic paraboloid



$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2} = C$$

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2} = C$$



POLL 1

$$\ln(u)$$
$$\sqrt[3]{v}$$

continuous $u > 0$
continuous $v \neq 0$

Evaluate

$$\lim_{(x,y) \rightarrow (8,e^3)} \ln(\sqrt[3]{xy}) = \ln(\sqrt[3]{8e^3})$$

$$= \ln(2e)$$

$$= \ln(2) + \ln(e)$$

$$= 1 + \ln(2)$$

a) $\ln(2)$

b) $\frac{1}{3}\ln(2)$

c) $1 + \ln(2)$

POLL 2

(Spring 2017 Exam 1 #6)

Evaluate:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x - 2y}{\sqrt{x^2 + y^2}}$$

a) 3

b) 1

c) The limit does not exist

$x=0$ $\lim_{y \rightarrow 0} \frac{-2y}{\sqrt{y^2}} =$

$\lim_{y \rightarrow 0} \frac{-2y}{|y|} \begin{cases} -2 & y > 0 \\ 2 & y < 0 \end{cases}$

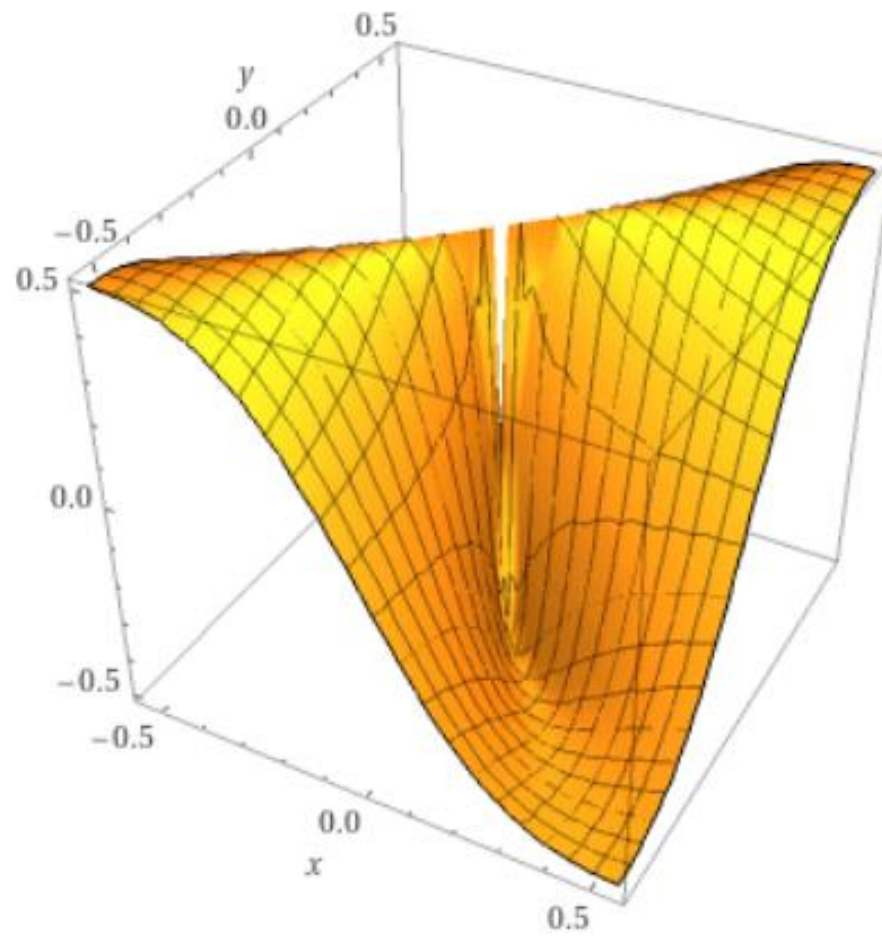
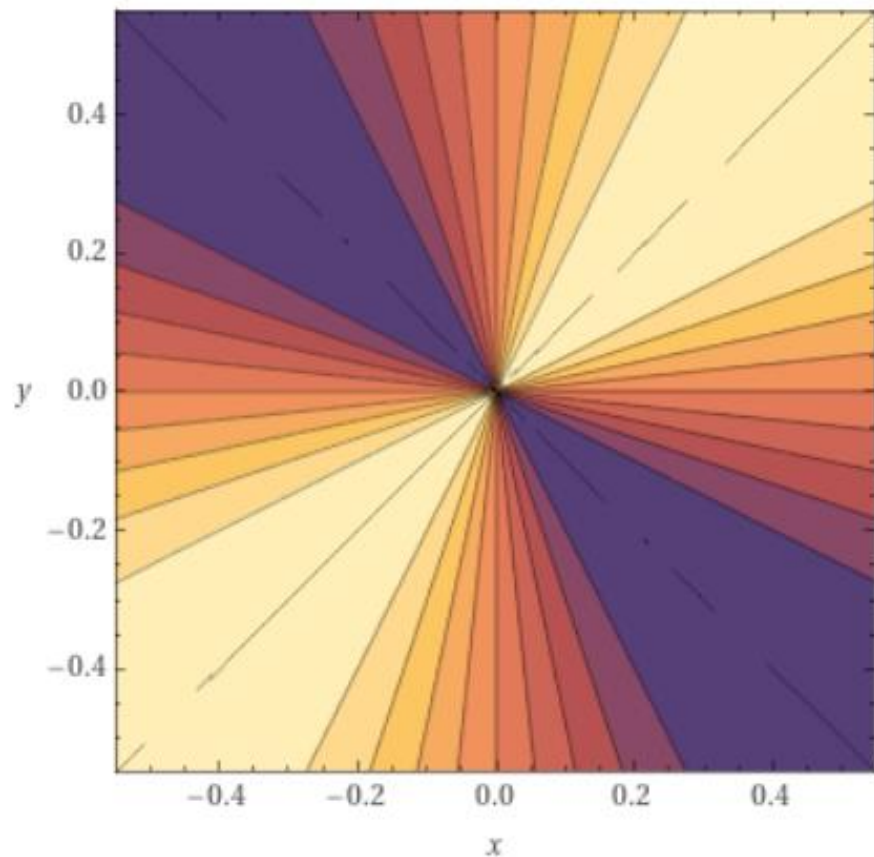
$y=0$ $\lim_{x \rightarrow 0} \frac{3x}{\sqrt{x^2}} = \begin{cases} +3 & x > 0 \\ -3 & x < 0 \end{cases}$

GRAPH

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

$$(x, y) \neq (0, 0)$$

$$(x, y) = (0, 0)$$



POLL 3

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(2x^2+y^2)}{4x^2+2y^2} = \lim_{u \rightarrow 0} \frac{\sin(u)}{2u}$$

$$u = 2x^2 + y^2$$

$$f(x, y) = \begin{cases} \frac{\sin(2x^2 + y^2)}{4x^2 + 2y^2}, & (x, y) \neq (0, 0) \\ b, & (x, y) = (0, 0) \end{cases}$$

Find the value of b that makes this function continuous.

a) 2

b) $\frac{1}{2}$

c) $\frac{1}{4}$

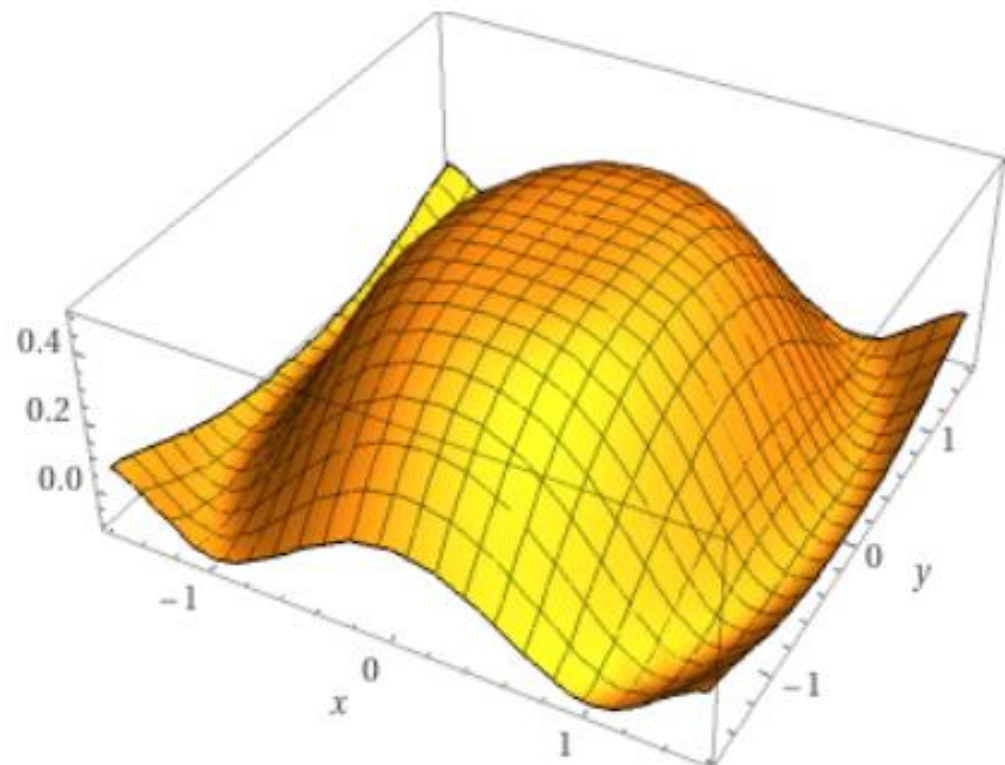
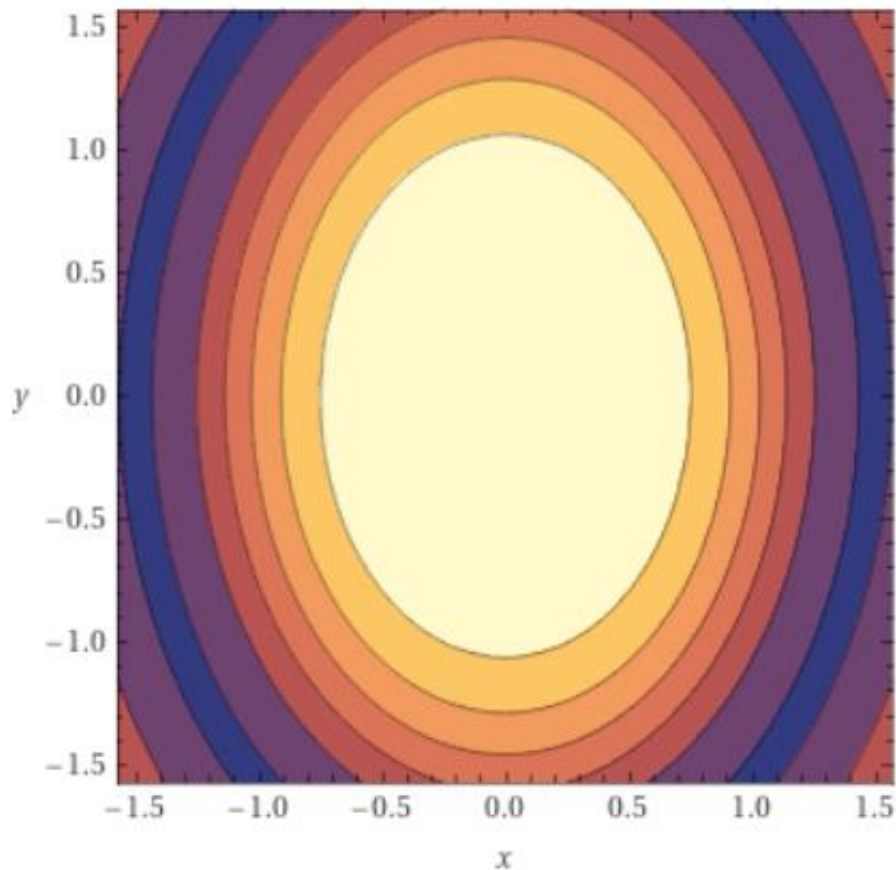
d) No such value of b exists

$$= \frac{1}{2} \lim_{u \rightarrow 0} \frac{\sin(u)}{u} = \frac{1}{2} \cdot 1$$

$$b = \frac{1}{2}$$

GRAPH

$$f(x, y) = \begin{cases} \frac{\sin(2x^2 + y^2)}{4x^2 + 2y^2}, & (x, y) \neq (0, 0) \\ \frac{1}{2}, & (x, y) = (0, 0) \end{cases}$$



MUDDIEST POINT

What was the muddiest point from today's lecture?

- a) Limits of functions of two variables
- b) Path
- c) Continuity
- d) None – understood everything today