LESSON 11

MA 26100·FALL 2023

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LESSON 11 – WARM UP

(Spring 23 Exam 1 #9) How should $f(0,0)$ be defined so that $f(x,y) = \frac{(x+y)^2}{x^2+y^2}$ is continuous at $(0,0)$?

along $y=x$ $\lim_{x\to 0} \frac{(x+x)^2}{x^2+x^2} = \lim_{x\to 0} \frac{4\cancel{x^2}}{2\cancel{x^2}} = 2$

a) $f(0,0) = 1$

b) $f(0,0) = 2$ along $y=0$ $\lim_{x\to 0} \frac{(x)^2}{x^2} = 1$ $\boxed{y=kx}$ $k=1$ $k=0$

c) $f(0,0) = 0$

d) Not possible to define $f(0,0)$ such that $f(x,y)$ is continuous at $(0,0)$

POLL 1

Let $f(x, y) = \sin(x^2y - 2y)$. Find $f_y = \frac{\partial f}{\partial y}$ at the point $(x, y) = (2,1)$

a) $2\cos(2)$

b) $\cos(2)$

c) $4\cos(2)$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\sin(x^2y - 2y)\right]$$

$$= \cos(x^2y - 2y)\frac{\partial}{\partial y}\left[x^2y - 2y\right]$$

$$= \cos(x^2y - 2y)\left[x^2 - 2\right]\Big|_{\substack{x=2\\ y=1}}$$

$$= \cos(2^2\cdot 1 - 2\cdot 1)\left[2^2 - 2\right]$$

$$2\cos(2)$$

POLL 2

(Spring 2017 Exam 1 #9)

If $f(x,y) = \ln\left(xy^2 + x\right)$, find f_{xy}

$$f_y = \frac{1}{\cancel{x}y^2+\cancel{x}} \cdot 2\cancel{x}y = \frac{y}{y^2+1}$$

a) 0

b) $\frac{-2y}{(xy^2+x)^2}$

c) $\frac{-2xy}{(xy^2+x)^2}$

$$f_{xy} = \frac{\partial}{\partial x}\left[f_y\right] = \frac{\partial}{\partial x}\left[\frac{y}{y^2+1}\right] = 0$$

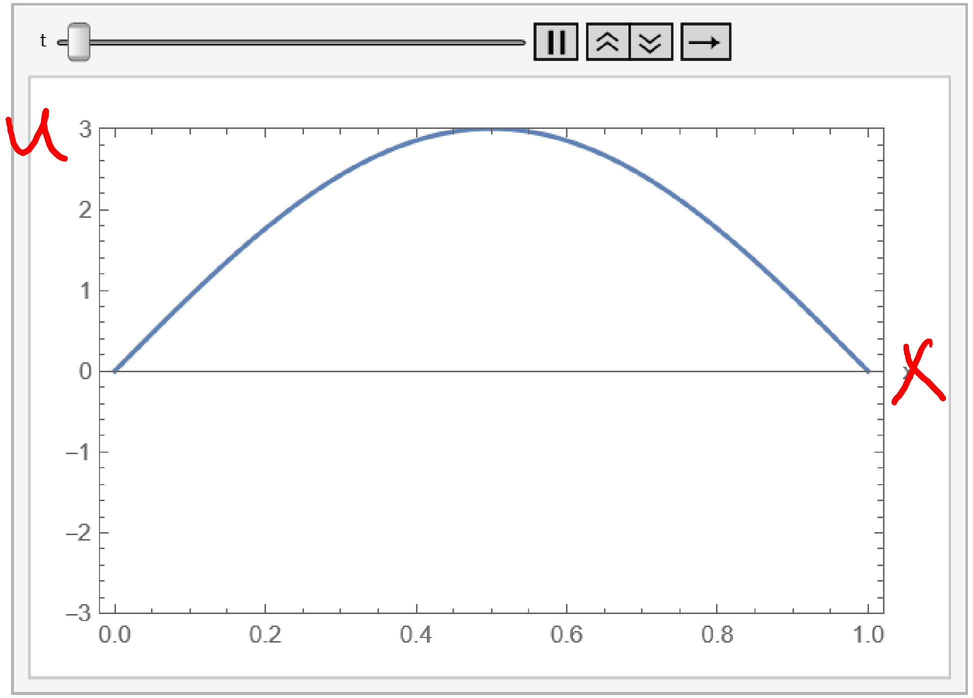
$$\boxed{f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial y}\right]}$$

$u(x=0, t) = 3\cos(2\pi t)\sin(0) = 0$

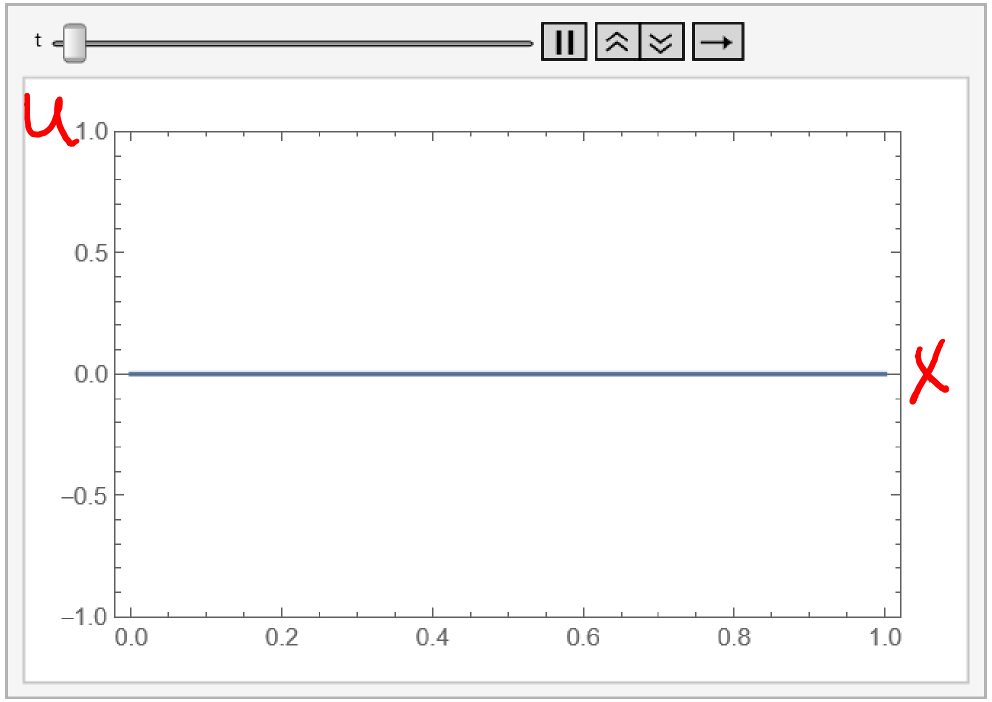
POLL 3

Which animation below depicts the motion of the guitar string (that solves the wave equation): $u(x,t) = \underbrace{3\cos(2\pi t)}_{\text{amplitude}}\sin(\pi x)$

a)



b)



POLL 4

$G'(t) = g(t)$

$$f(x,y) = \int_{2x+1}^{y^2} g(t)dt = G(y^2) - G(2x+1)$$

For some function $g(t)$. Find $\frac{\partial f}{\partial y}$

$a)\ g(y^2)$

$b)\ g(2y)$

$c)\ 2y\, g(y^2)$

$d)\ g'(2y)$

$\frac{\partial}{\partial y}\left[G(y^2) - G(2x+1)\right]$

$G'(y^2)(2y) = 2y\, g(y^2)$

MUDDIEST POINT

What was the muddiest point from today’s lecture?

a) First order partial derivatives
b) Second order partial derivatives
c) Mixed partial derivatives
d) Solutions to partial differential equations
e) None – understood everything today