LESSON 12

MA 26100·FALL 2023

DR. HOOD

LESSON 12 – WARM UP

(Spring 23 Exam 1 #10) If $f(x, y) = xy^2 + ye^{x^2} + 5$, find f_{xx}.

$f_x = y^2 + y(2x)e^{x^2}$

$f_{xx} = 0 + 2ye^{x^2} + y(2x)^2 e^{x^2}$

$= 2ye^{x^2}(1 + 2x^2)$

a) $ye^{x^2}(1 + x^2)$

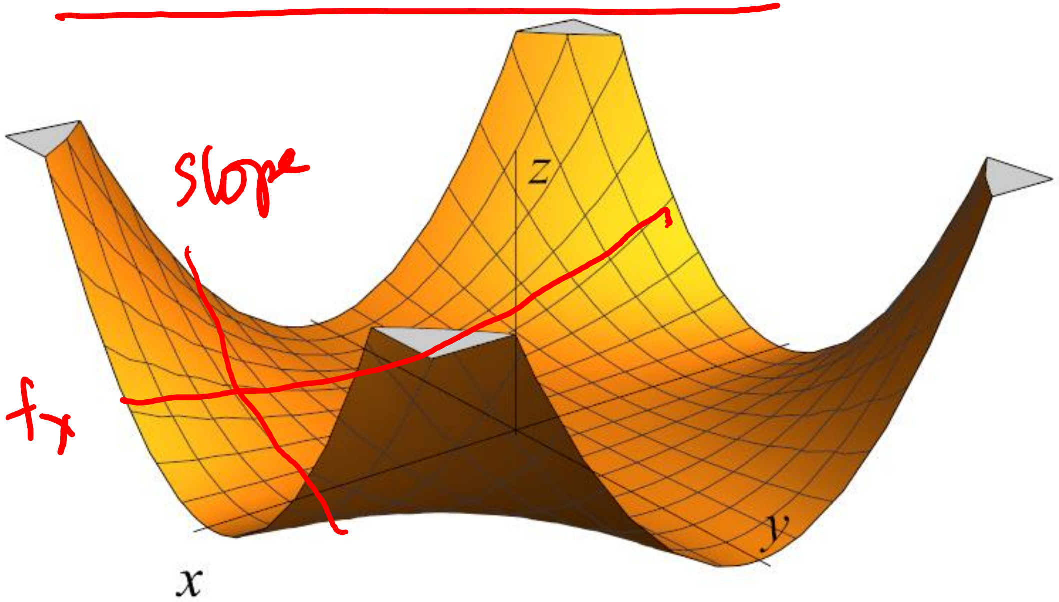
b) $2ye^{x^2}(1 + 2x^2)$

c) $2xe^{x^2} + y^2$

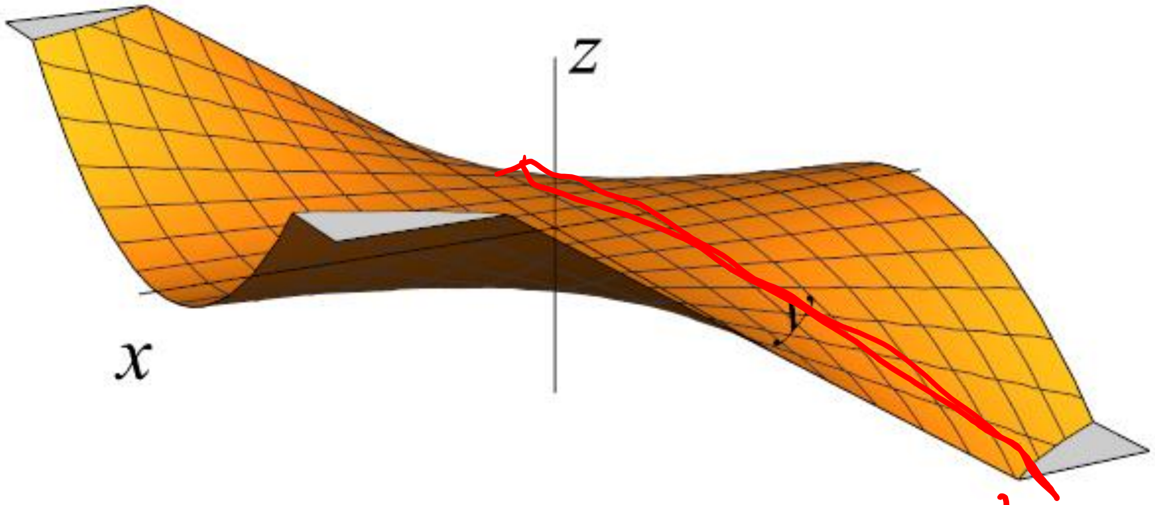
d) $2xye^{x^2}$

PARTIAL DERIVATIVES

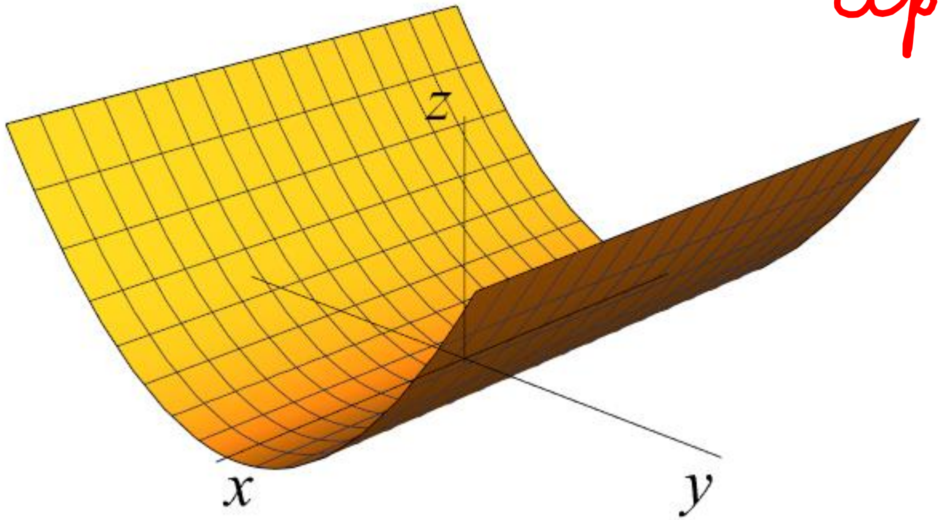
$z = f(x, y) = x^2y^2$



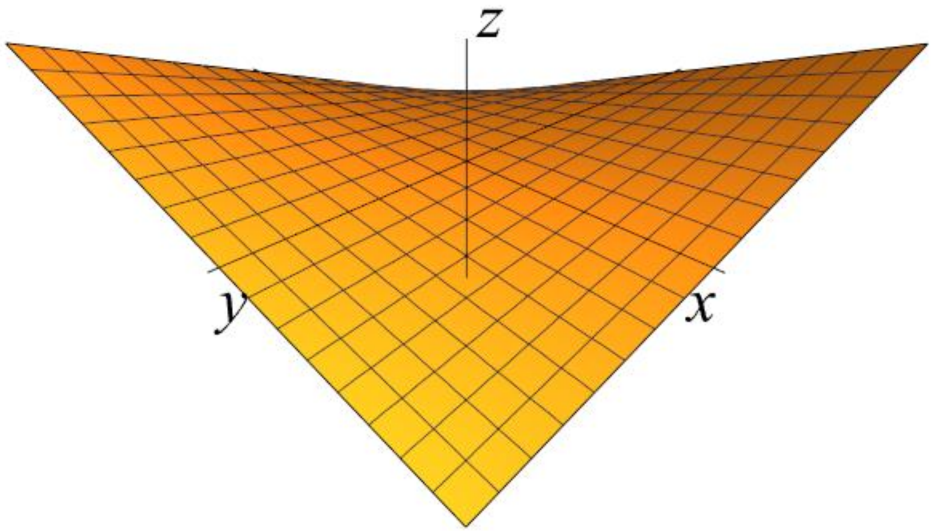
$f_x = 2xy^2$ slope in x



$f_{xx} = 2y^2 \geq 0$ concave up



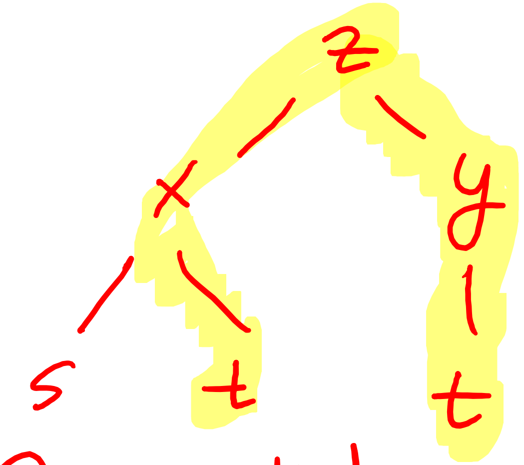
$f_{xy} = 4xy$



POLL 1

$\frac{\partial z}{\partial x} = xy^2$ $\quad \frac{\partial x}{\partial t} = 2$

$\frac{\partial z}{\partial y} = x^2y + y^2$ $\quad \frac{dy}{dt} = 2t$



(Similar to Spring 17 Exam 1 #8) Let

$$z = f(x, y) = \frac{1}{2}x^2y^2 + \frac{1}{3}y^3$$

@ $s=2$ $t=1$

$x = 2+2\cdot 1 = 4$

$x = s + 2t$ and $y = t^2$. Find z_t at the point $(s, t) = (2,1)$ $y = 1^2 = 1$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

$$= (xy^2)\frac{\partial x}{\partial t} + (x^2y + y^2)\frac{dy}{dt}$$

$$= (xy^2)\,2 + (x^2y + y^2)(2t)$$

$x=4$, $y=1$, $s=2$, $t=1$

$$= 4\cdot 1^2\cdot 2 + (4^2\cdot 1 + 1^2)\cdot 2\cdot 1 = 8 + 17\cdot 2 = 34 + 8$$

$\boxed{42}$

a) 14

b) 8

c) 42

d) 32

POLL 2

Suppose $z = z(x, y)$ is defined implicitly by

$$e^{xyz} - 2 = 0$$

Find $\frac{\partial z}{\partial y}$.

$$\frac{\partial}{\partial y}\left[e^{xyz} - 2\right] = 0$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\cancel{x}z\cancel{e^{xyz}}}{\cancel{x}y\cancel{e^{xyz}}} = -\frac{z}{y}$$

a) $\frac{-z}{y}$

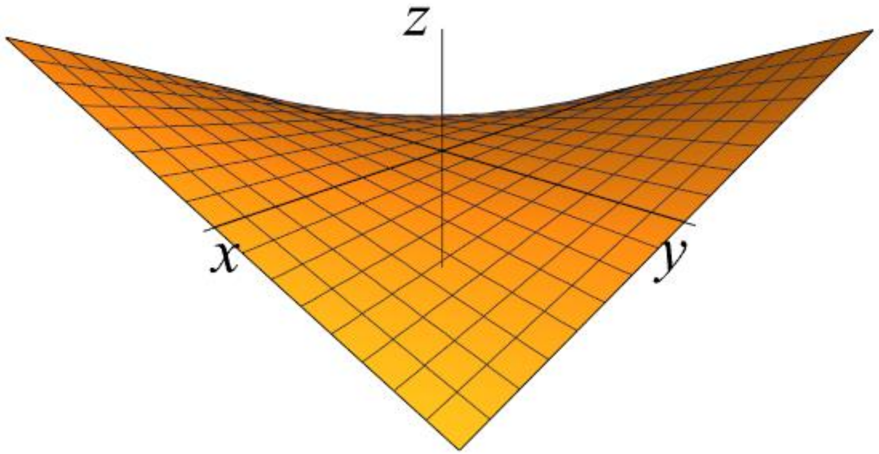
b) xye^{xyz}

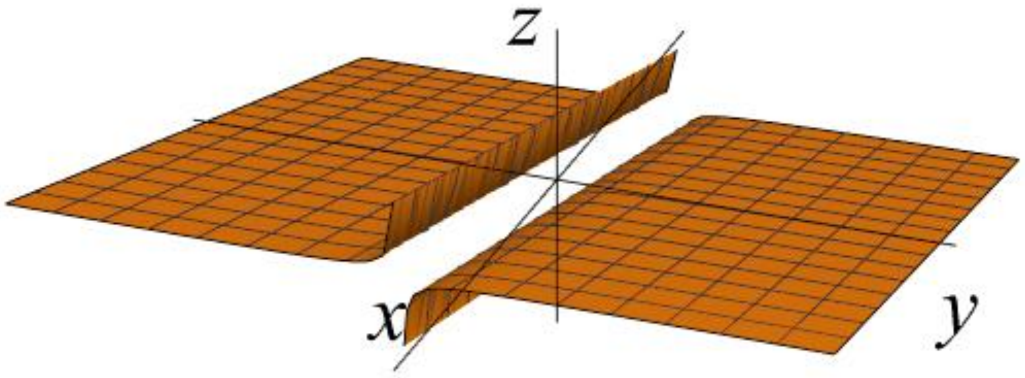
c) $\frac{y}{z}$

POLL 2 - GRAPHS

$e^{xyz} - 2 = 0$

$$\frac{\partial z}{\partial y} = -\frac{z}{y}$$





MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Chain Rule with 1 dependent variable
b) Chain Rule with 2 dependent variables
c) Implicit Differentiation
d) None – understood everything today