LESSON 13

MA 26100·FALL 2023

DR. HOOD

WARM UP

Implicit Differentiation

(Fall 22 Exam 1 #10)

$F(x,y) = 0 = (x^2+y^2)^3 - 8x^2y^2$

Consider $\left(x^2+y^2\right)^3 = 8x^2y^2$. Find the derivative $\frac{dy}{dx}$ at the point $(x,y) = (-1,1)$.

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{\left[3(x^2+y^2)^2(2x) - 2\cdot 8xy^2\right]}{\left[3(x^2+y^2)^2(2y) - 2\cdot 8x^2y\right]}\Bigg|_{\substack{x=-1\\y=+1}}$$

$$= -\frac{\left[3(2)^2(-2) + 16\cdot(+1)\right]}{3(2)^2(2) - 16}$$

$$= 1$$

a) -1

b) 1

c) 0

d) The derivative does not exist

e) $\frac{1}{2}$

LESSON 13

POLL 1

(Fall 22 Exam 1 #10) $D_{\vec{u}} f = \frac{\partial f}{\partial x}(a,b)\, u_1 + \frac{\partial f}{\partial y}(a,b)\, u_2$

Compute the directional derivative $D_{\mathbf{u}}f$ of

$$f(x,y) = xy + x^3$$

at the point $(1,2)$ in the direction of $\langle 1,-1\rangle = \vec{v}$

$\frac{\partial f}{\partial x} = y + 3x^2 \Big|_{x=1,\, y=2} = 2 + 3 = 5$

$\frac{\partial f}{\partial y} = x \Big|_{x=1,\, y=2} = 1$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1,-1\rangle}{\sqrt{2}}$

$D_{\vec{u}} f = 5 \cdot \frac{1}{\sqrt{2}} + 1\left(\frac{-1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

a) 4

b) $2\sqrt{3}$

c) $2\sqrt{2}$

d) $3\sqrt{2}$

POLL 2

(Spring 23 Exam 1 #11)

Find the directional derivative of

$$f(x,y) = x^3 e^{-2y}$$

in the direction of greatest increase of f at $x = 1$ and $y = 0$.

steepest ascent

$a)$ $3\hat{\mathbf{i}}$

$b)$ $3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$c)$ $\sqrt{5}$

$d)$ $\sqrt{13}$

$$\frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3,-2\rangle}{\sqrt{9+4}} = \frac{\langle 3,-2\rangle}{\sqrt{13}}$$

$$\vec{\nabla} f = \langle 3x^2 e^{-2y}, -2x^3 e^{-2y}\rangle \Big|_{\substack{x=1\\y=0}}$$

$$\vec{\nabla} f = \vec{v} = \langle 3, -2\rangle \longrightarrow \vec{u} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}$$

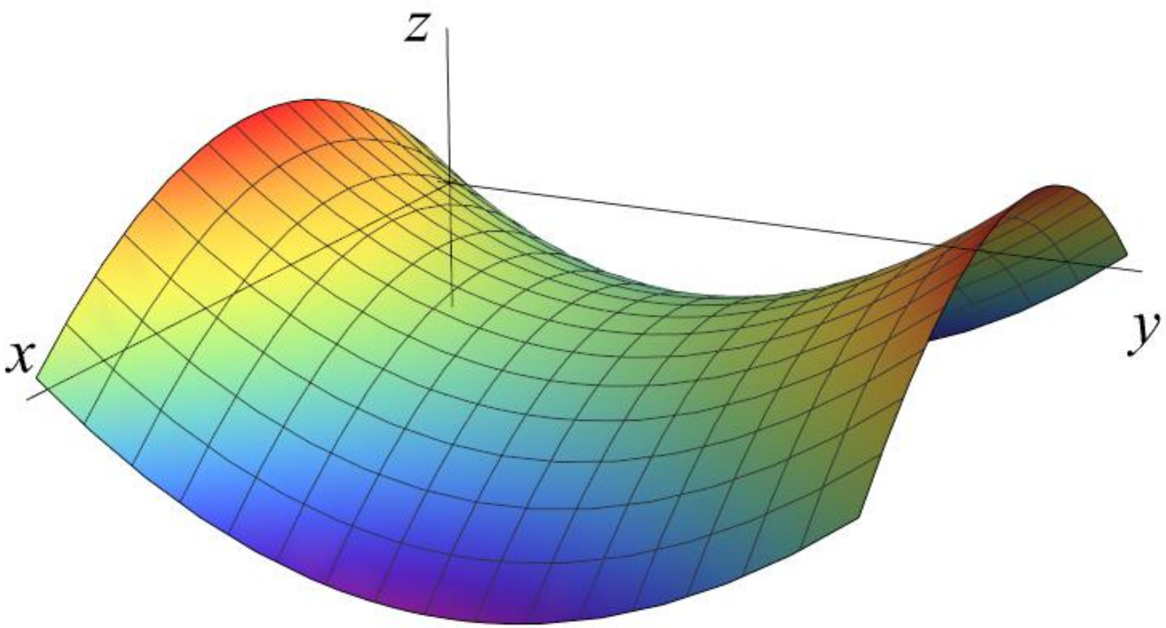
$$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u} = \vec{\nabla} f \cdot \frac{\vec{\nabla} f}{|\vec{\nabla} f|}$$

$$= \frac{|\vec{\nabla} f|^2}{|\vec{\nabla} f|} = |\vec{\nabla} f| = \sqrt{13}$$

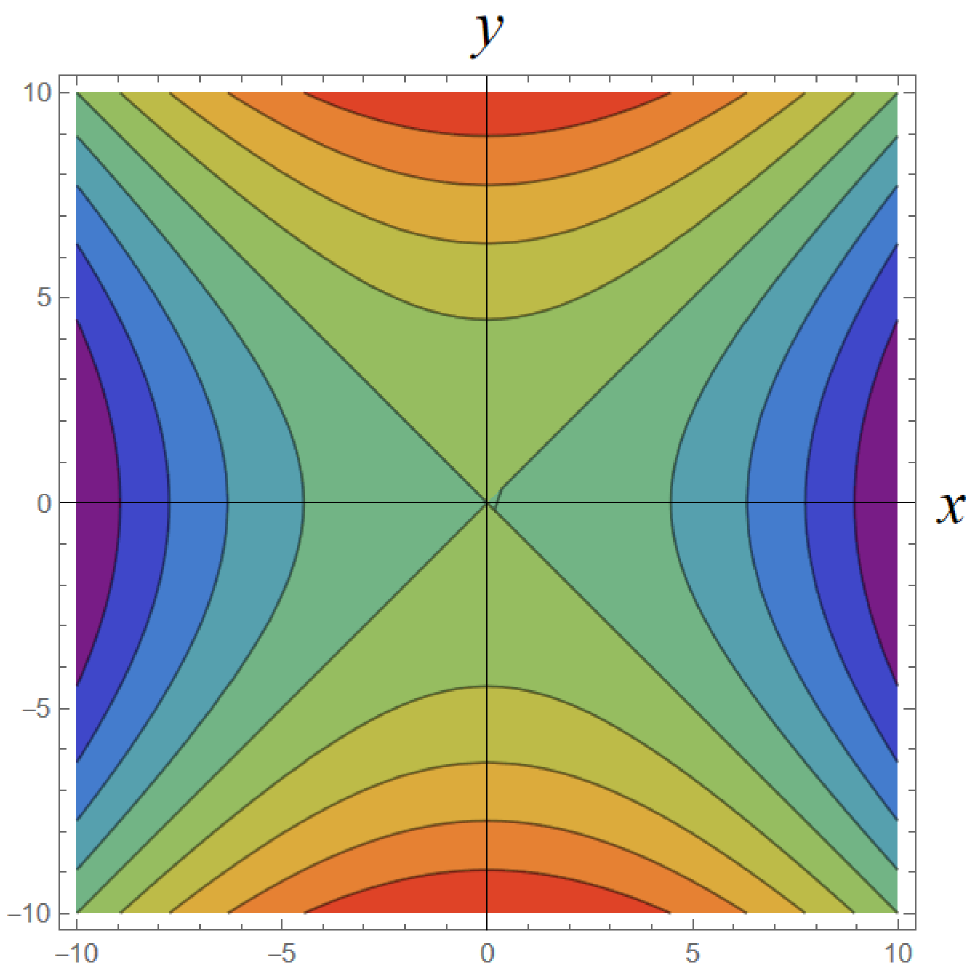
Consider the hyperbolic paraboloid:

$$z = -x^2 + y^2$$

Surface:



Level Curves:



GRADIENT

Consider the hyperbolic paraboloid:

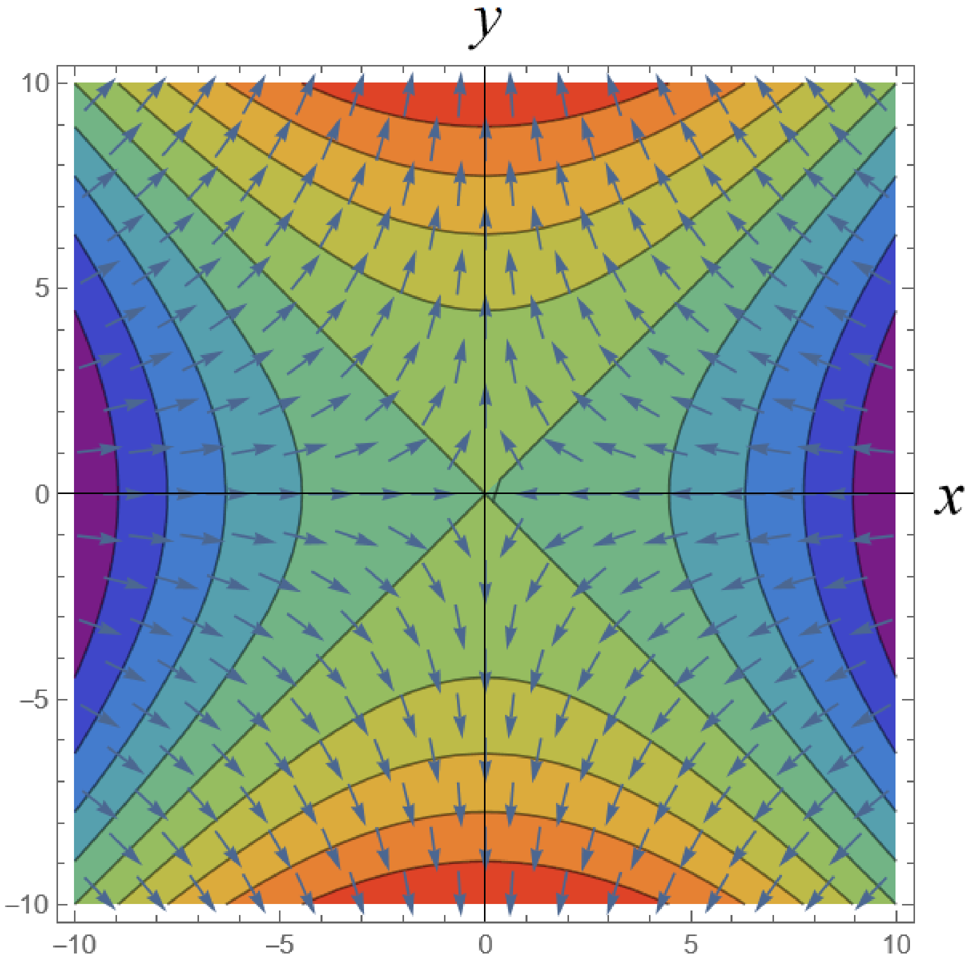
$$z = f(x, y) = -x^2 + y^2$$

Gradient:

$\nabla f = ?$ $\langle -2x, 2y \rangle$

Direction of steepest ascent

Level Curves:



MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Directional Derivative
b) Gradient
c) Direction of Steepest Ascent
d) None – understood everything today