LESSON 14

MA 26100·FALL 2023

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WARM UP

(Fall 14 Exam 1 #8)

For which direction $\vec{\mathbf{u}}$ will the directional derivative of $f(x, y) = xy^{-2}$ at the point $(2, 1)$ have the value 0?

a) $\langle 1, -4 \rangle$

b) $\left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$

c) $\langle 4, 1 \rangle$

d) $\left\langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \right\rangle$

$0 = D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$

$\vec{\nabla} f = \langle y^{-2}, -2xy^{-3} \rangle \Big|_{x=2,\ y=1}$

$= \langle 1, -4 \rangle$

$\vec{u} = \langle u_1, u_2 \rangle$

$|\vec{u}| = 1$

$\sqrt{u_1^2 + u_2^2} = 1$

$\vec{\nabla} f \cdot \vec{u} = 0$

$\langle 1, -4 \rangle \cdot \langle u_1, u_2 \rangle = 0$

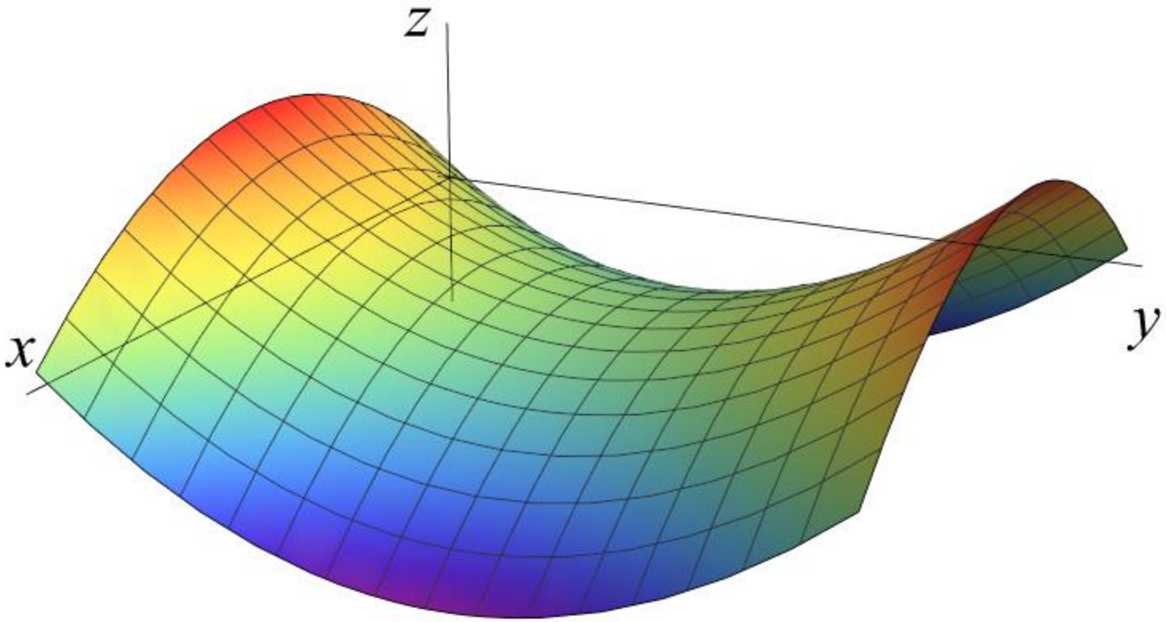
$u_1 - 4u_2 = 0 \qquad u_1 = 4u_2$

LESSON 14

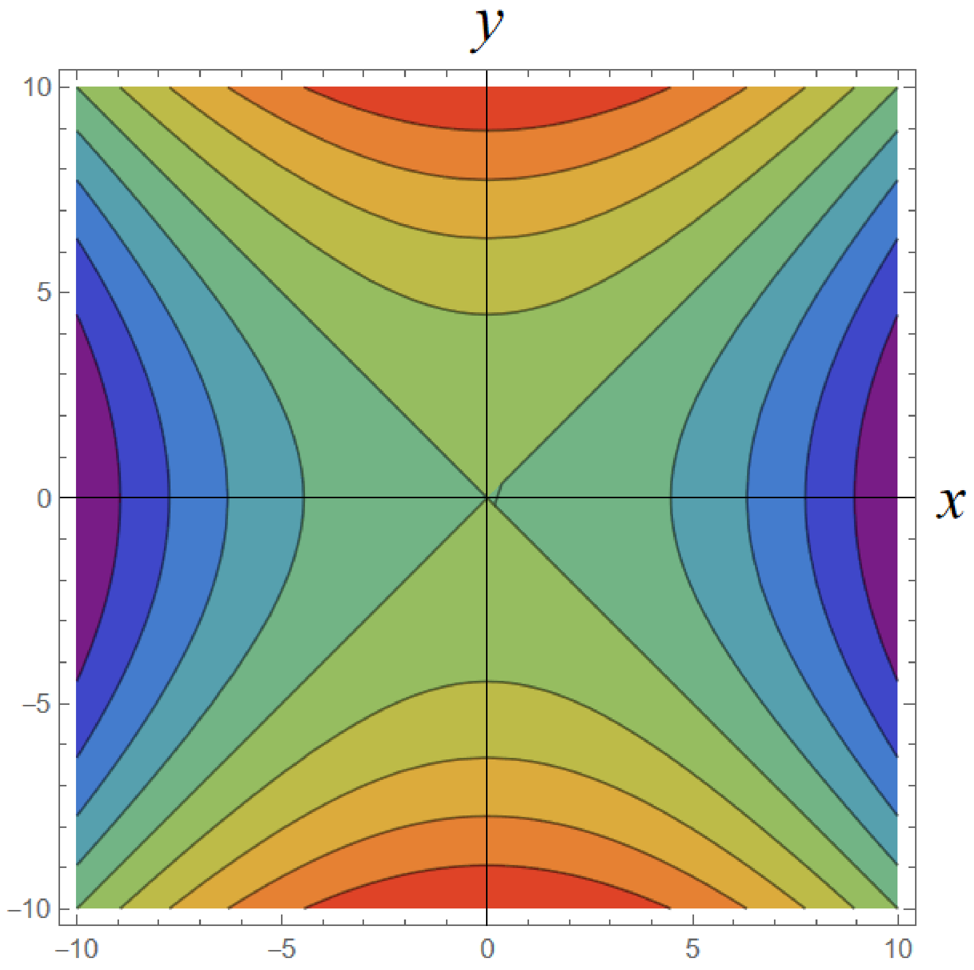
Consider the hyperbolic paraboloid:

$$z = -x^2 + y^2$$

Surface:



Level Curves:



GRADIENT

Consider the hyperbolic paraboloid:

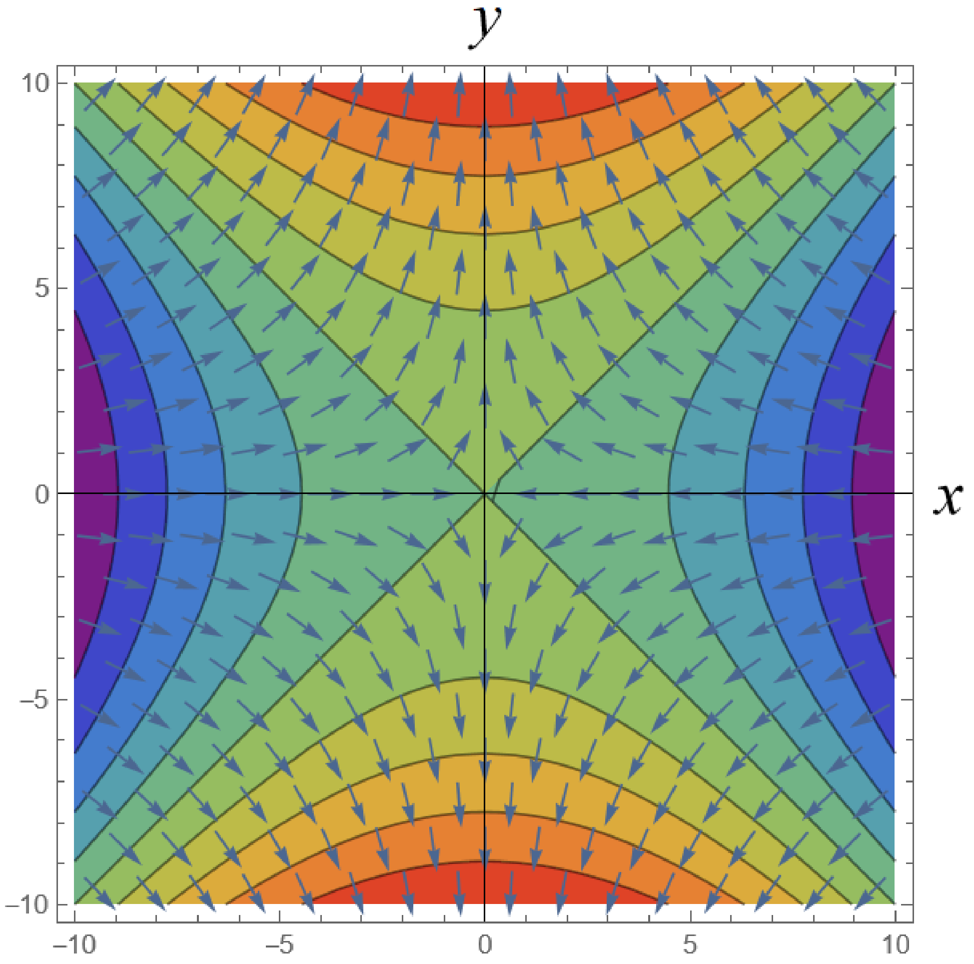
$$z = f(x, y) = -x^2 + y^2$$

Gradient:

$\nabla f = \langle -2x, 2y \rangle$

Direction of steepest ascent

Level Curves:



Consider the hyperbolic paraboloid:

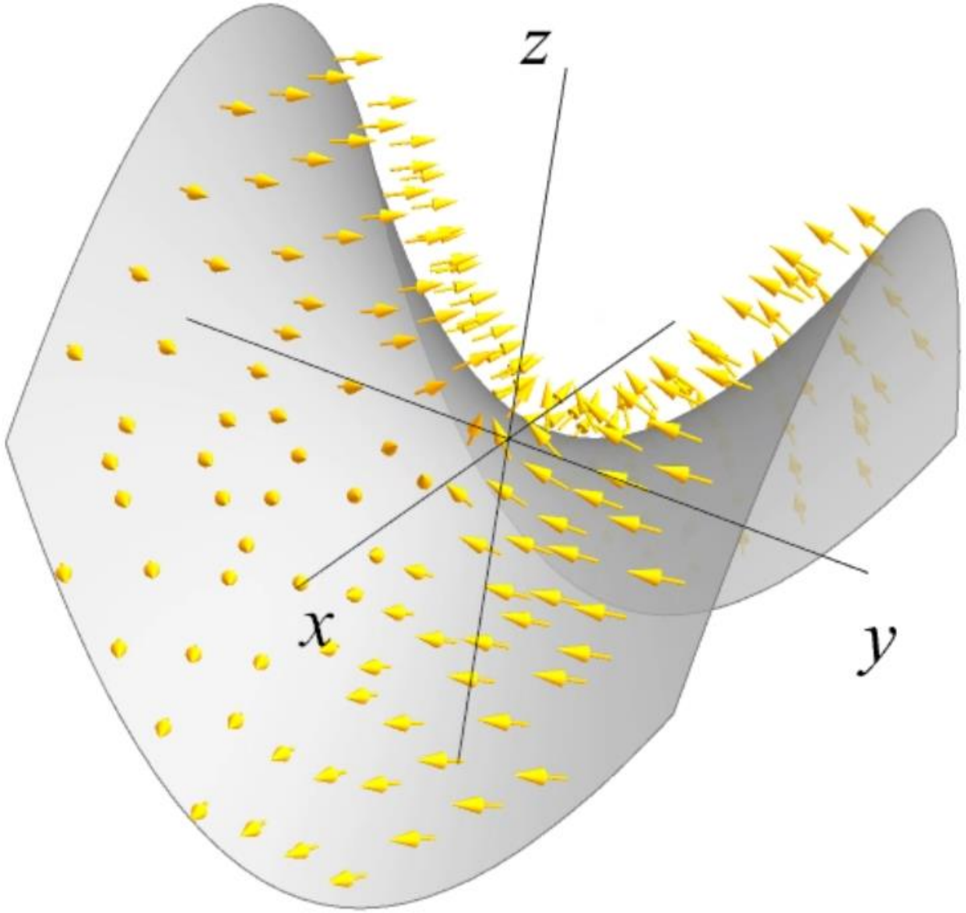
$$F(x, y, z) = z + x^2 - y^2 = 0$$

Gradient:

$$\nabla F = \langle 2x, -2y, 1 \rangle$$

$\nabla F(a,b,c)$ is normal to the surface at (a,b,c)

Gradient is normal to surface:



POLL 1

(Fall 17 Exam 1 #11)

Find the tangent plane to the level surface

$$xy^2z^3 = 12$$

at the point $(3, 2, 1)$

$a)\ x + y + z = 6$

$b)\ 3x + 2y + z = 14$

$c)\ x + 3y + 9z = 18$

$\vec{n} = \langle 1, 3, 9 \rangle$

$F(x,y,z) = xy^2z^3 - 12 = 0$

$\vec{\nabla} F \cdot \langle x-a, y-b, z-c \rangle = 0$

$F_x(x-a) + F_y(y-b) + F_z(z-c) = 0$

$\nabla F = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle \Big|_{x=3,\ y=2,\ z=1}$

$= \langle 4, 12, 36 \rangle$

$= 4 \langle 1, 3, 9 \rangle$

$\langle 1, 3, 9 \rangle \cdot \langle x-3, y-2, z-1 \rangle = 0$

POLL 2

Find the linear approximation of

$$f(x, y) = e^x \cos(y)$$

near the point $(0,0)$.

$a)\ L(x, y) = 1 + x$

$b)\ L(x, y) = 1 + x - y$

$c)\ L(x, y) = x + y$

$d)\ L(x, y) = 1 + x + y$

POLL 3

(Fall 15 Exam 1 #7)

Consider the function $f(x, y, z) = xyz$. Which of the following is true?

1) $df = x\,dx + y\,dy + z\,dz$
2) If $\Delta x = \Delta y = \Delta z = 0.2$, then the error estimated by using differentials at $(1, 2, 1)$ is $\Delta f = 1$
3) Its linear approximation at $(1, 1, 1)$ is $L = x + y + z - 2$

a) All are true
b) Only 2) and 3) are true
c) Only 3) is true

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Directional Derivative
b) Gradient
c) Direction of Steepest Ascent
d) None – understood everything today