LESSON 15

MA 26100·FALL 2023

DR. HOOD

WARM UP

LESSON 15

(Fall 22 Exam 1 #11)

Horizontal Plane: $z = k$

Tangent Plane:

$z = f(a,b) + \cancel{f_x\Big|_{\substack{x=a\\y=b}}}\,(\cancel{x-a}) + \cancel{f_y\Big|_{\substack{x=a\\y=b}}}\,(\cancel{y-b})$

Consider the surface

$$z = 3 - x^2 - y^2 + 6y$$

Find all the at the points on the surface at which the tangent plane is horizontal.

$\nabla f = \langle 0,0 \rangle = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$

$\langle -2x, -2y+6 \rangle = \langle 0,0 \rangle$

$x=0 \qquad y=3$

a) $(0, 3, 12)$

b) $(1, 2, 3)$

c) $(3, 2, 3)$

d) $(12, 3, 0)$

EXAM 1 INFORMATION

- Tuesday, October 3, 2023 at Time: 8:00pm – 9:00pm
- Location: ELLT 116 and Loeb Playhouse.
  - Seating chart: https://www.math.purdue.edu/academic/courses/semester/202410/ma26100/resources/ma261_exam_seating_chart.pdf
- Lessons covered on the exam: Lessons 1 – 16.
  - Study Guide: https://www.math.purdue.edu/~kthood/docs/MA261_Fall2023/exam1_study_guide_ma261_fa23-merged.pdf

MAXIMA & MINIMA

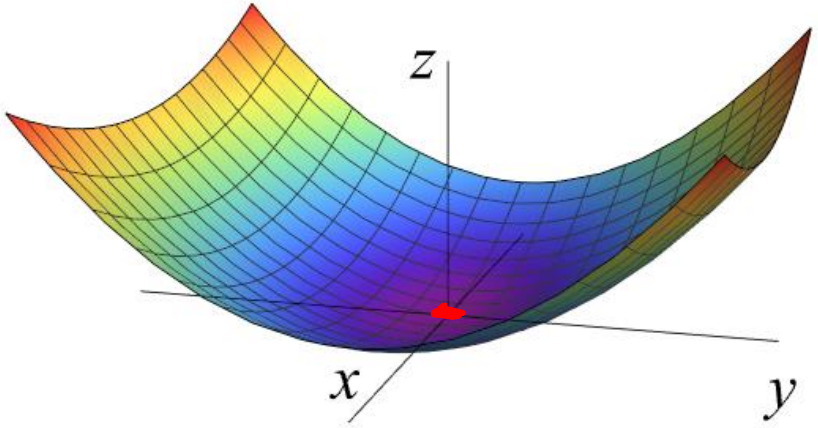
$D > 0$

$f_{xx}, f_{yy} > 0$

Elliptic Parabola

$z = x^2 + 3y^2$

Critical Point: (0,0)



local min

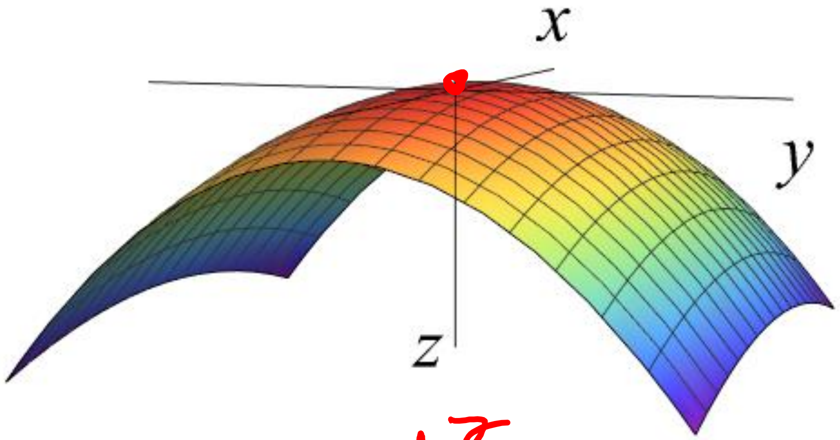
$D > 0$

$f_{xx} < 0$

Elliptic Parabola

$z = 3 - x^2 - 5y^2$

Critical Point: (0,0)



local max

$D < 0$

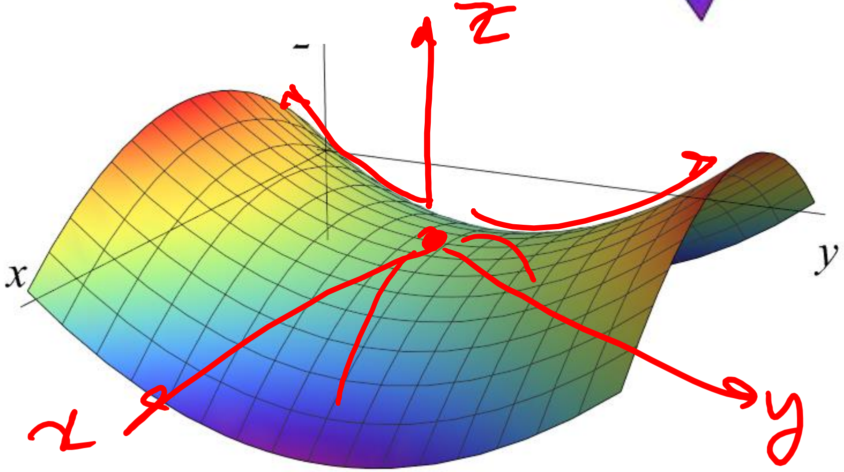
$f_{xx} = -2$

$f_{yy} = 2$

Hyperbolic paraboloid

$z = -x^2 + y^2$

Critical Point: (0,0)



saddle point

POLL 1

Let

$$F(x,y) = 3 - x^2 + xy - 5y^2$$

The origin $(0,0)$ is a critical point. Classify which type of critical point it is.

$f_x = -2x + y$

$f_{xx} = -2$ $\qquad f_{xy} = 1$

$f_y = x - 10y$

$f_{yy} = -10$

$D = (-2)(-10) - 1^2 = 19 > 0$

$f_{xx} < 0$ local max

a) Local max

b) Local min

c) Saddle point

d) Not enough information

POLL 2

(Fall 2017 Exam 1 #8)

$f_x = 2x + 4y - 2 = 0 \longrightarrow x = 1 - 2y$

$f_y = y^2 + 4x - 13 = 0 \longleftarrow y^2 + 4(1-2y) - 13 = 0$

Find the point (x, y) at which f has a local minimum. $y = -1, 9$

$$f(x,y) = \frac{1}{3}y^3 + x^2 + 4xy - 2x - 13y + 7$$

$f_{xx} = 2$ $\quad f_{xy} = 4$

$f_{yx} = 4$ $\quad f_{yy} = 2y$

a) $(1, -13)$

b) ~~$(3, -1)$~~

c) $(-17, 9)$

$\boxed{@\ y = -1}$ $D = 2 \cdot (-2) - 4^2 < 0$ saddle point

$\boxed{y = 9}$ $D = 2 \cdot (18) - 4^2 > 0$ local min

$f_{xx} > 0$

POLL 3

Classify the critical point $(0, 0)$ of the function

$$f(x, y) = x^3 y$$

According to the 2nd Deriv. Test

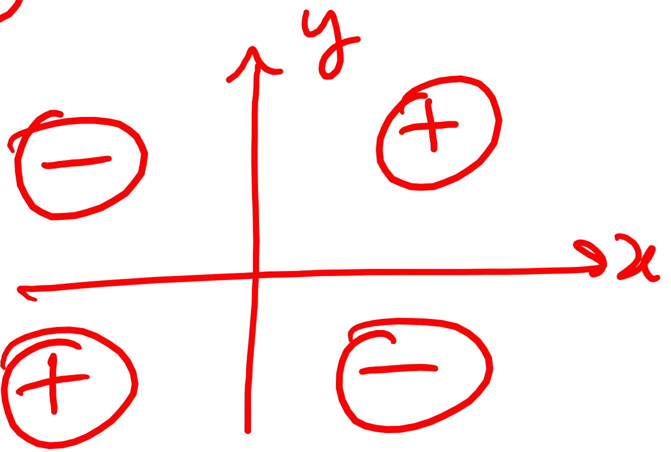
a) Local max
b) Local min
c) Saddle point
d) Not enough information

$f_x = 3x^2 y$ $\quad f_y = x^3$

$f_{xx} = 6xy$ $\quad f_{xy} = 3x^2$

$f_{yy} = 0$

$D = 0 \cdot 0 - 0 \cdot 0 = 0$



sign of $f(x,y)$

saddle point

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Critical Point
b) Discriminant
c) Second Derivative Test
d) Saddle point
e) None – understood everything today