Lesson 15 - 9/25/2023 - 4:30pm

**15.7: Maximum & Minimum Problems - Part 1**

Calc 1: Q: Where does $f(x)$ have local max / min?

1. Find the critical point(s) $c$ where
   $f'(c) = 0$
   $f'(c)$ undefined
   } $c$ is a possible candidate for local max or min

2. Second Derivative Test

   $f''(c) > 0$ + + local min

   $f''(c) < 0$ − − local max

   $f''(c) = 0$ not enough information

extrema - max or min

Calc 3:

Q: where does $f(x,y)$ have a local min/max?

1. Find the critical points where tangent plane is horizontal
   $\begin{cases} f_x(c_1, c_2) = 0 = f_y(c_1, c_2) \\ f_x(c_1, c_2) \text{ or } f_y(c_1, c_2) \text{ be undefined} \end{cases}$

Example: $f(x,y) = (x-2)^2 + 3(y-1)^2$

find the critical point(s)

$f_x = 2(x-2) = 0 \rightarrow x = 2$

$f_y = 3 \cdot 2(y-1) = 0 \rightarrow y = 1$

$\boxed{\text{critical point } (2,1)}$

Q: How do we determine if $(2,1)$ is a local max or min?

**Q:** How do we [illegible] local max or min?

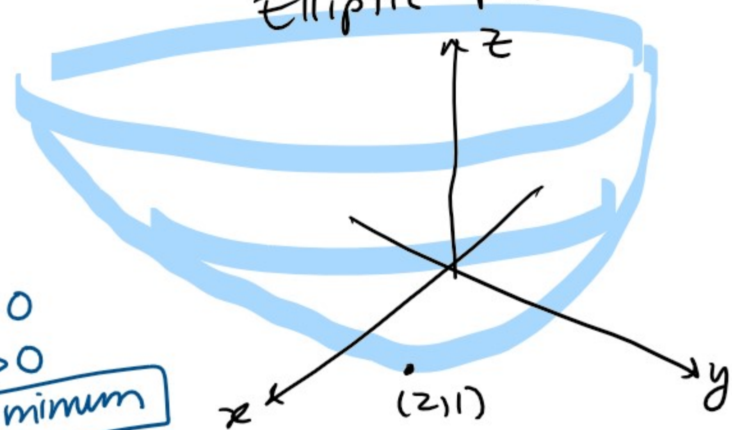
$f$ has 4 second derivatives:

$f_{xx} = 2 \qquad f_{yx} = 0$

$f_{xy} = 0 \qquad f_{yy} = 6$

$z = f(x,y) = (x-2)^2 + 3(y-1)^2$

Elliptic Parabola




$f_{xx} > 0$ ++ along x-axis minimum

$f_{yy} > 0$ ++ along y-axis minimum

$f(2,1) = 0$

$D > 0$
$f_{xx} > 0$
$\boxed{\text{minimum}}$

**Def:** The discriminant $D$ of $f(x,y)$ is:

$$D = f_{xx} f_{yy} - f_{xy} f_{yx} \Big|_{\substack{x = c_1 \\ y = c_2}}$$

$D = 2 \cdot 6 - 0^2 = 12 > 0$

All of functions $f_{xy} = f_{yx}$

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

**Second Derivative Test:**

Let $(c_1, c_2)$ be a critical point of $f(x,y)$

Let $D = f_{xx} f_{yy} - (f_{xy})^2 \Big|_{\substack{x = c_1 \\ y = c_2}}$

| $D$ | $f_{xx}/f_{yy}$ | $(c_1, c_2)$ |
|---|---|---|
| + | + | local min |
| + | − | local max |
| | | saddle point |

| | | |
|---|---|---|
| + | − | local |
| − | anything | saddle point |
| 0 | anything | Not enough info |

Q: why do need the discriminant

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

Ex: $f(x,y) = \frac{x^2}{2} + 2xy + \frac{y^2}{2}$

$$\left.\begin{aligned} f_x &= x + 2y = 0 \\ f_y &= 2x + y = 0 \end{aligned}\right\}$$ (0,0) satisfies so its a critical pt.

Discriminant

$f_{xx} = 1$ $\quad f_{xy} = 2$

$f_{yx} = 2$ $\quad f_{yy} = 1$

$$D = f_{xx} f_{yy} - (f_{xy})^2 = 1 \cdot 1 - 2^2 = -3$$

saddle point