



LESSON 16

MA 26100-FALL 2023

DR. HOOD

(Spring 23 Exam 2 #11)

Find the relative extrema of

$$f(x, y) = -\frac{2}{3}x^3 + 4xy - 2y^2 + 1$$

- ~~a) (3, 2) local max~~
- b) (1, 3) local min
- ~~c) (2, 3) local max~~
- d) (2, 2) local min
- e) (2, 2) local max**

C.p. : $f_x = 0 = -2x^2 + 4y = 0$

$$f_y = 0 = 4x - 4y = 0$$

$$x = y$$

critical pts : (2, 2) and (0, 0)

2nd Deriv Test

$$f_{xx} = -4x \big|_{x=2} = -8$$

$$f_{xy} = 4$$

$$f_{yy} = -4$$

$$D = (-8)(-4) - 4^2 > 0$$

$f_{xx} < 0$

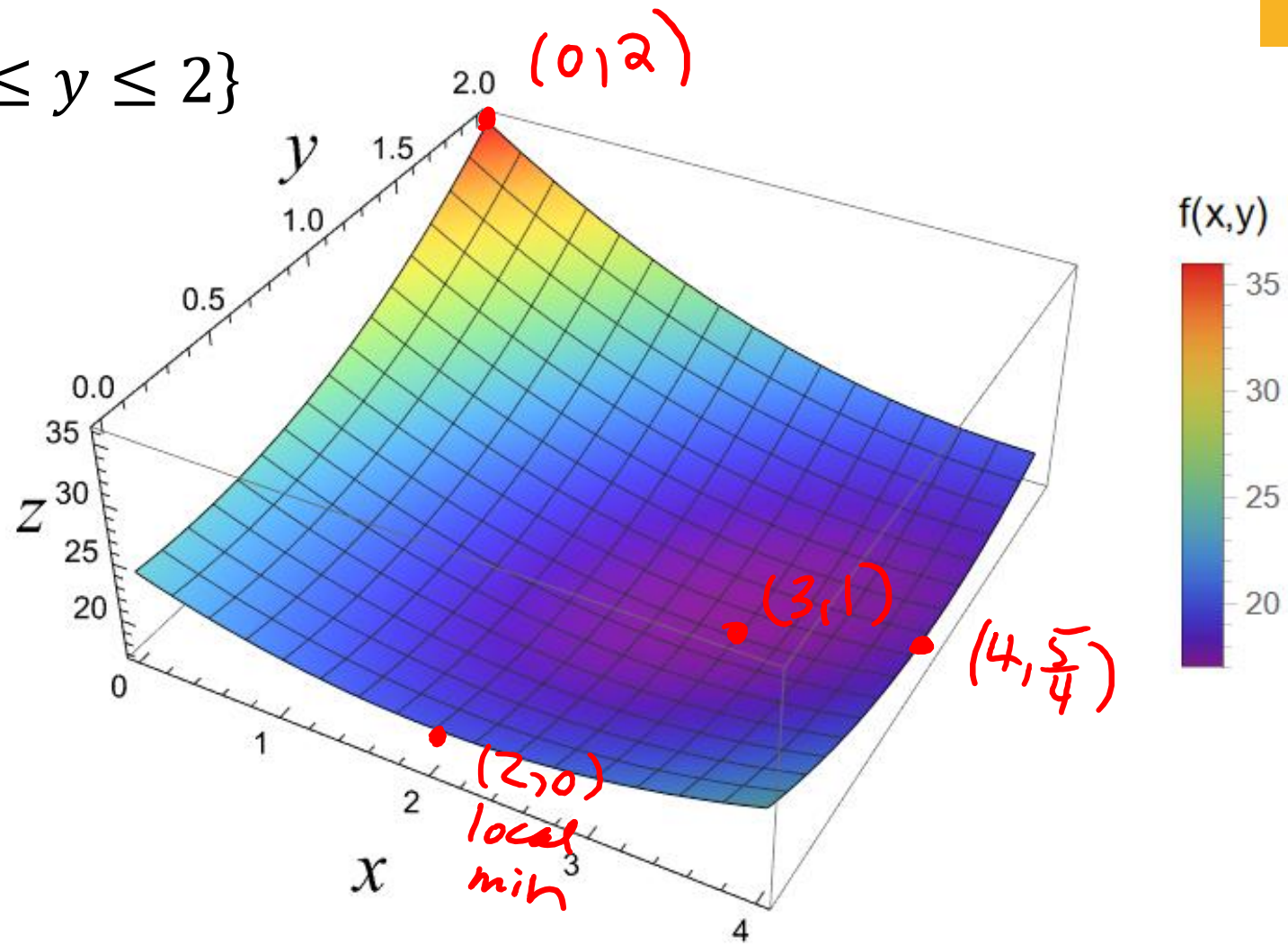
ABSOLUTE EXTREMA

Find the absolute extrema of

$$f(x, y) = x^2 - 2xy + 4y^2 - 4x - 2y + 24$$

Over the domain

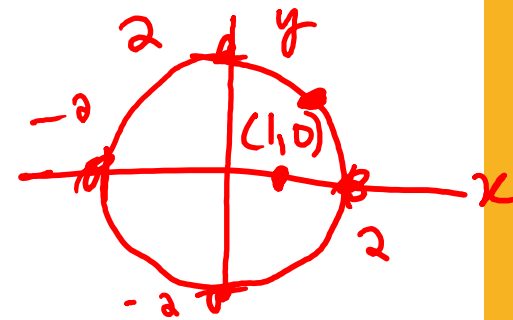
$$D = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq 2\}$$



The function

$$f(1,0) = 9 \leftarrow \text{global max}$$

$$f(x,y) = 9 - x^2 - y^2 + x$$



On the domain $D = \{(x,y) : x^2 + y^2 \leq 4\}$ has a critical point at $(1,0)$. Are there any global extrema on the boundary of the domain? (Hint: check when $x^2 + y^2 = 4$)

$$x = 2 \cos t, \quad y = 2 \sin t \\ 0 \leq t \leq 2\pi$$

a) Global max

$$f(t) = 9 - 4 \cos^2 t - 4 \sin^2 t + 2 \cos t \\ = 5 + 2 \cos t$$

b) Global min

$$f'(t) = -2 \sin(t) = 0 \\ t = 0 \quad t = \pi$$

c) Global max and Global min

d) No global extrema on the boundary of D

$$\text{@ } t=0 \quad f(2,0) = 9 - 4 + 2 = 7$$

$$\left. \begin{array}{l} \text{@ } t=\pi \\ f(-2,0) = 3 \\ \text{global min} \end{array} \right\}$$

(Spring 2023 Exam 2 #4)

The x -coordinate of the point on the plane $z = x + y$ that is closest to the point $(1, 1, 1)$ is equal to:

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $\frac{3}{4}$

d) $\frac{5}{4}$

e) $\frac{1}{4}$

Classify the critical point $(0, 0)$ of the function

$$f(x, y) = x^3y$$

- a) Local max
- b) Local min
- c) Saddle point
- d) Not enough information

MUDDIEST POINT

What was the muddiest point from today's lecture?

- a) Boundary of Domain
- b) Evaluating on the Boundary
- c) Minimize distance to a plane
- d) None – understood everything today