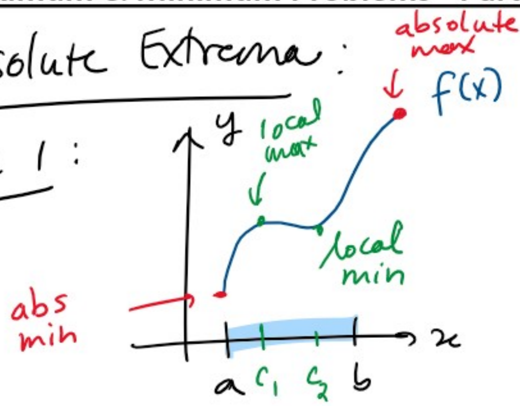
Lesson 16 - 9/25/2023 - 4:30pm

15.7: Maximum & Minimum Problems - Part 2

Absolute Extrema:

Calc 1:

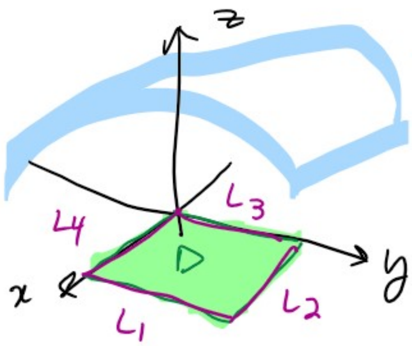


Find absolute max/min on domain $[a,b]$

check the critical points
$f'(c)=0 \quad f'(c)$ DNE

also check the endpoints
$x=a$ and $x=b$

Calc 3: Find the absolute extrema of $f(x,y)$ over the domain D in the xy-plane



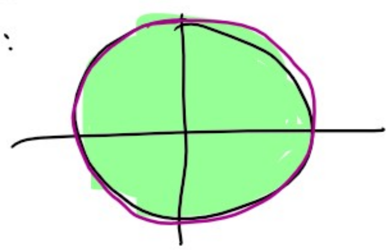
check the critical points
$(c_1, c_2) \leftarrow$ make sure inside D

check the boundary of the Domain D

Boundary: L_1, L_2, L_3, L_4

Types of Domains:

Ex:



$D = \{(x,y) : x^2+y^2=1\}$

Boundary: circle of radius 1

$x^2+y^2=1$

$y = \pm\sqrt{1-x^2}$

parameterize by t

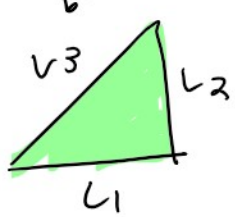
$x=\cos(t)$

$y=\sin(t)$

$\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Triangle



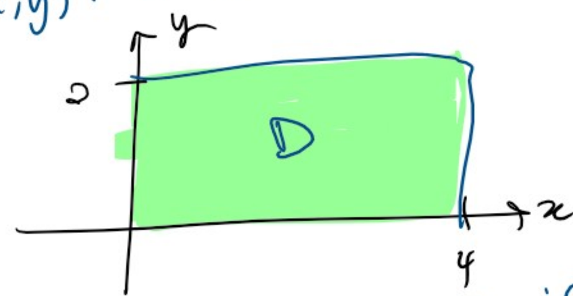
L_1

Example: Find the absolute extrema of

$$f(x,y) = x^2 - 2xy + 4y^2 - 4x - 2y + 24$$

on the domain

$$D = \{ (x,y) : 0 \le x \le 4 \text{ and } 0 \le y \le 2 \}$$



$\boxed{\text{Step 1}}$ Find critical points $\rightarrow$ check if inside D

$f_x = 0 = 2x - 2y - 4 = 0$

$f_y = 0 = -2x + 8y - 2 = 0$

$$\begin{array}{r} x - y = 2 \\ + \quad -x + 4y = 1 \\ \hline 3y = 3 \\ y = 1 \end{array}$$

critical point $(3,1)$ is inside D

$x - y = 2$
$x - 1 = 2$
$x = 3$

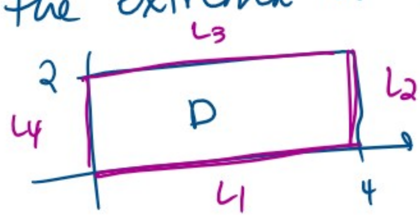
2nd Deriv Test:

$f_{xx} = 2 \quad f_{xy} = -2 \quad f_{yy} = 8$

$D = 2 \cdot 8 - (-2)^2 = 16 - 4 = 12 > 0$

$f_{xx} > 0$ $\boxed{\text{local min}}$

$\boxed{\text{Step 2}}$ Find the extrema of on boundary



L_1: $y = 0$ $\quad (0 \le x \le 4)$

L_2: $x = 4$ $\quad (0 \le y \le 2)$

$(0 \le x \le 4)$

L_2: $x = 4$ $(0 \le y \le 2)$

L_3: $y = 2$ $(0 \le x \le 4)$

L_4: $x = 0$ $(0 \le y \le 2)$

$\boxed{\text{check } L_1}$ $y = 0 \longrightarrow$ plug into f

$$f(x,y) = x^2 - \cancel{2xy} + \cancel{4y^2} - 4x - \cancel{2y} + 24 \Big|_{y=0}$$

$$= x^2 - 4x + 24 \quad \text{on} \quad 0 \le x \le 4$$

$f' = 2x - 4 \longrightarrow c = 2$

$f'' = 2 > 0$ ++ local min

$(2,0) \rightarrow$ local min

check end points too

$(0,0)$ and $(4,0)$

no c.p.

Do this for L_2, L_3, L_4

(x,y)	desc.	$f(x,y)$	type
$(3,1)$	c.p. off in D	17	local min abs min
$(2,0)$	c.p. of L_1	20	local min
$(4,\frac{5}{4})$	c.p. of L_2	17.75	local min
$(0,0)$	corner	24	
$(4,0)$	corner	24	
$(0,2)$	corner	36	abs. max
$(4,2)$	corner	20	

Optimization:

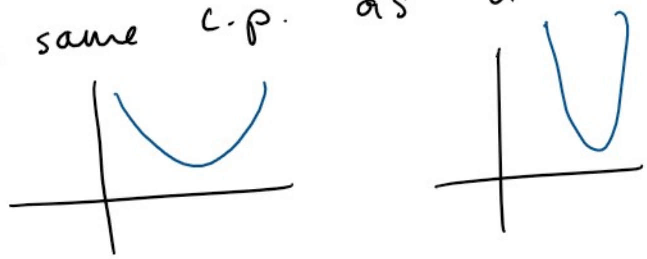
Ex: Find the point in the plane $z = x+y$ that is closest to the point $(1,1,1)$

minimize distance

$$d(x,y,z) = \sqrt{(x-1)^2 + (y-1)^2 + (z-1)^2}$$

minimize d^2 (b/c $d \geq 0$)

d^2 has same c.p. as d



Find (x,y,z) that minimizes $f = (x-1)^2 + (y-1)^2 + (z-1)^2$

where $z = x+y$

$$f(x,y) = (x-1)^2 + (y-1)^2 + (x+y-1)^2$$

Find c.p.

$$f_x = 2(x-1) + 2(x+y-1) = 0$$
$$4x + 2y - 4 = 0$$

$$f_y = 2(y-1) + 2(x+y-1) = 0$$
$$2x + 4y - 4 = 0$$

$$\begin{aligned} 2x + y &= 2 \\ - \quad x + 2y &= 2 \\ \hline x - y &= 0 \end{aligned} \rightarrow x = y$$

$2x + y = 2$

$3x = 2$

$x = \frac{2}{3} = y$

$z = x + y = \frac{4}{3}$

critical point: $\boxed{\left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}\right)}$

2nd Deriv Test

critical point: [illegible]

2nd Deriv Test

$f_{xx} = 4$ $\quad f_{xy} = 2$ $\quad f_{yy} = 4$

$D = 4^2 - 2^2 > 0$ $\quad f_{xx} > 0$ $\quad \boxed{\text{min}}$