LESSON 17

MA 26100·FALL 2023

DR. HOOD

WARM UP

(Fall 16 Exam 2 #2)

Critical points: $f_x = 0 = 2x(y^3 - y)$

$f_y = 0 = x^2(3y^2 - 1) + 2y$

According to the Second Derivative Test, the critical points of the function

$(0,0)$ $(1,-1)$ $(-1,-1)$

$$f(x,y) = x^2(y^3 - y) + y^2$$

Are:

a) Two saddle points

b) Two saddle points, one undetermined

c) One max, one min, one undetermined

$2x(y^3 - y) = 0$

$2x = 0$ or $y^3 - y = 0$

$x = 0$

$y(y^2 - 1) = 0$

$y = 0$ $y^2 = 1$

$y = \pm 1$

$\boxed{x=0 \quad y=0}$ $f_y = 0 \rightarrow 0 + 2y = 0 \rightarrow y = 0$

$\boxed{y=-1}$ $f_y = 0 \rightarrow x^2(3-1) - 2 = 0$

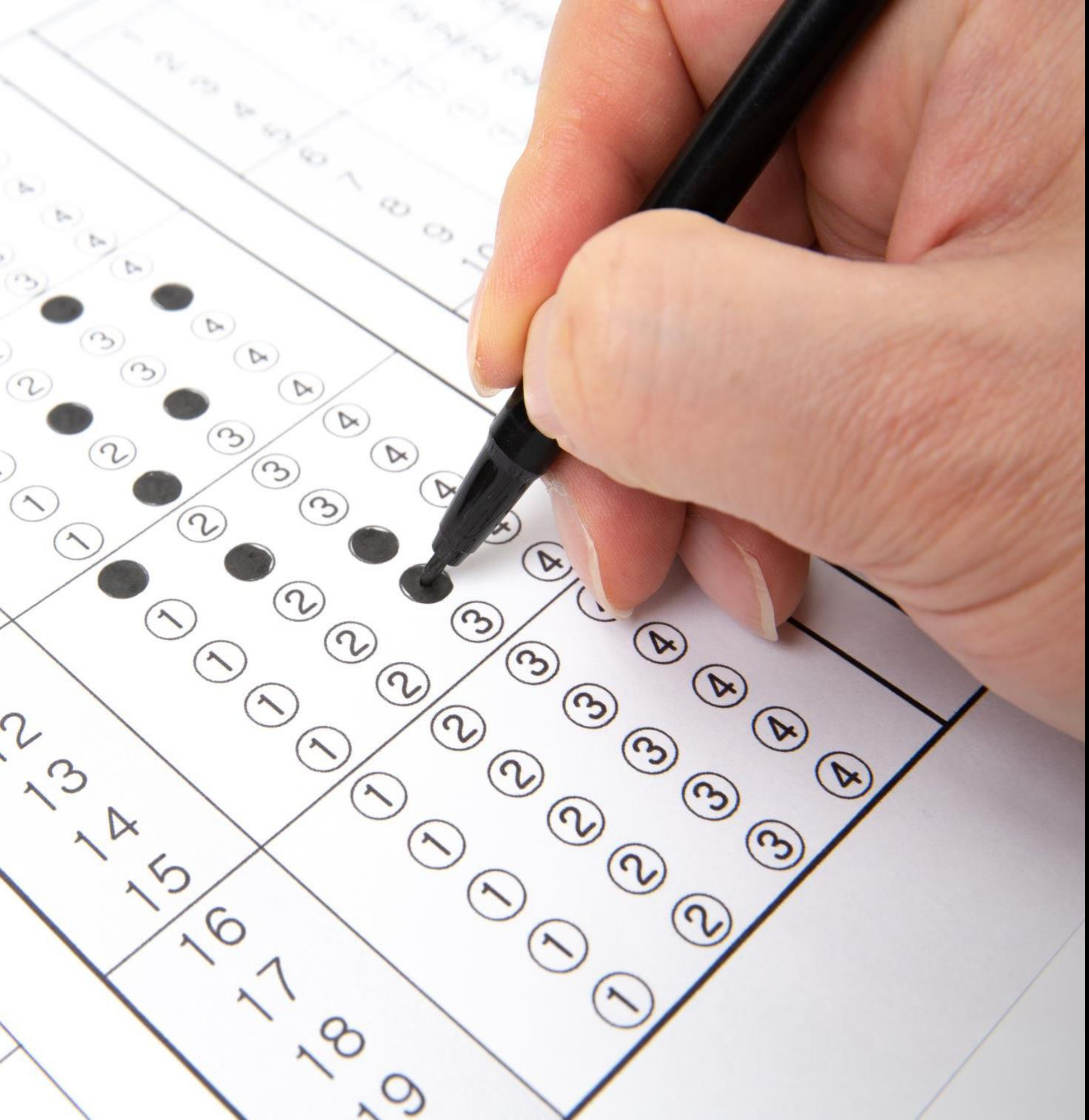
$2x^2 = 2$

$x = \pm 1$

$\boxed{x=1 \quad y=-1}$

$\boxed{x=-1 \quad y=-1}$

LESSON 17



EXAM 1 REVIEW

Supplemental Instruction

Monday, October 2, 2023

6:30pm – 8:20pm

UC 114

CONSTRAINT

Find the absolute extrema of

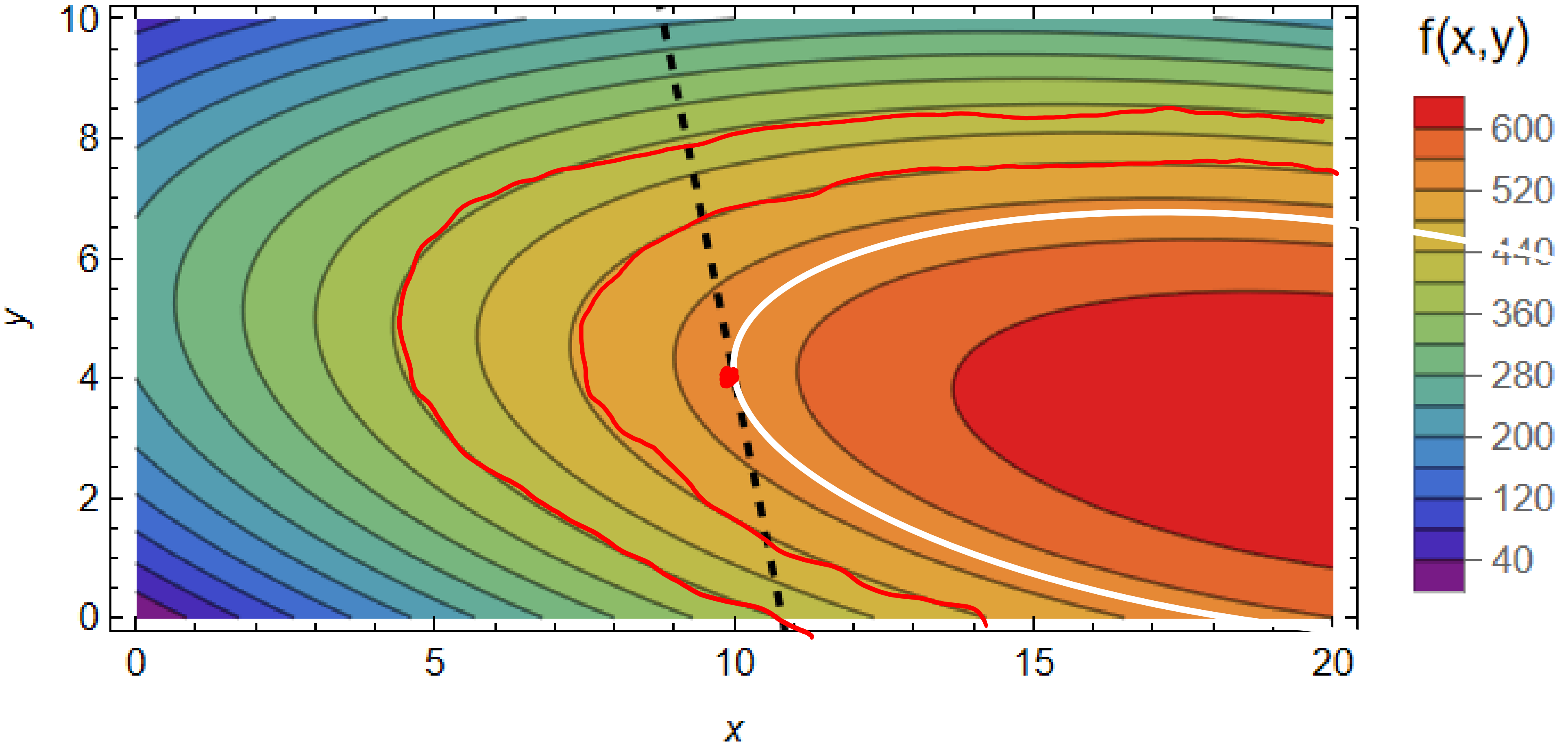
$$f(x,y) = -x^2 - 2xy - 9y^2 + 48x + 96y$$

Subject to the constraint

$$5x + y - 54 = 0$$

constraint $g(x,y)=0$

level curves $f(x,y)=k$



POLL 1

Let $g(x, y) = x^2 + y^2 - 1 = 0$ be the constraint curve. Let $(a, b) = (1,0)$ and the parameterize the constraint curve by $x(s) = \cos(s)$ and $y(s) = \sin(s)$. What is the relationship between the tangent vector $\vec{\boldsymbol{T}}(0)$ and the gradient $\vec{\nabla} g(a, b)$?

$\vec{r}(s) = \langle x(s), y(s) \rangle$

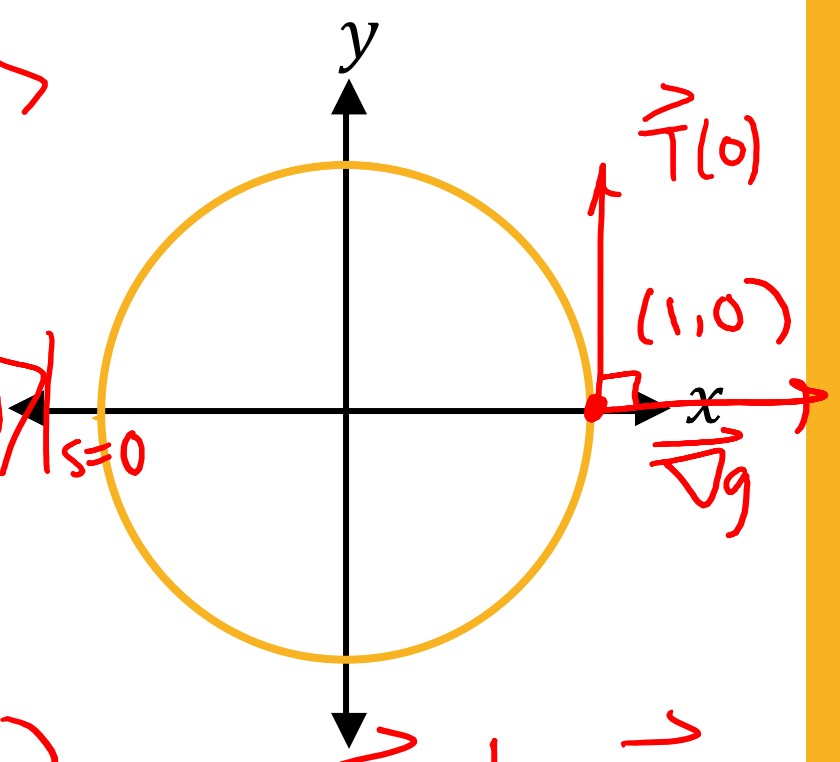
$\vec{T} = \vec{r}\,'(s) = \langle x', y' \rangle$

a) $\vec{\boldsymbol{T}}(0)$ is parallel to $\vec{\nabla} g(a, b)$

b) $\vec{\boldsymbol{T}}(0)$ is orthogonal to $\vec{\nabla} g(a, b)$

$\vec{T}(0) = \left\langle \frac{dx}{ds}, \frac{dy}{ds} \right\rangle \Big|_{s=0} = \langle -\sin(s), \cos(s) \rangle \Big|_{s=0}$

$= \langle 0, 1 \rangle$



$\vec{\nabla} g = \langle 2x, 2y \rangle \Big|_{\substack{x=1 \\ y=0}} = \langle 2, 0 \rangle$

$\vec{T} \perp \vec{\nabla} g$

POLL 2

(Spring 2023 Exam 2 #3)

Find the maximum of $f(x, y, z) = x + y + z$ subject to the constraint $(x-1)^2+y^2+z^2 = 1$.

a) $1+\sqrt{3}$

b) $1-\sqrt{3}$

c) $\sqrt{3}$

POLL 3

Find numbers x, y, z whose sum is 27 and the sum of whose squares are as small as possible.

$x+y+z=27 \leftarrow g$

$\min\ x^2+y^2+z^2 = f(x,y,z)$

$a)\ x = y = 10$ and $z = 7$

$b)\ x = y = z = 9$

$c)\ x = 8, y = 9, z = 10$

MUDDIEST POINT

What was the muddiest point from today’s lecture?

a) Lagrange multiplier
b) Gradient of the constraint curve
c) Solving the system of equations
d) None – understood everything today