LESSON 18

MA 26100·FALL 2023

DR. HOOD

WARM UP

LESSON 18

(Spring 20 Exam 2 #7)

Find the absolute maximum, M, and the absolute minimum, m, of the function $f(x,y) = x + y$ subject to the constraint $x^2 - xy + y^2 = 1$. $= g$

a) M = 2 and m=-2

b) M = 1 and m=-1

c) M = 1 and m=-4

d) M = 2 and m=-1

$\nabla f = \lambda \nabla g$

$\langle 1, 1 \rangle = \lambda \langle 2x - y, -x + 2y \rangle$

$1 = \lambda(2x - y)$

$1 = \cancel{\lambda}(-x + 2y) = \cancel{\lambda}(2x - y)$

$3y = 3x \rightarrow y = x$

$x^2 - xy + y^2 = 1$

$\cancel{x^2} - \cancel{x^2} + x^2 = 1$

$y = x = \pm 1$

$f(1,1) = 2 = M$

$f(-1,-1) = -2 = m$

ANNOUNCEMENTS

- No Class on Friday Oct 6 → No OH
- October Break
  - No class on Monday Oct 9
  - No recitation on Tuesday Oct 10
- Expect to return exam 1 scores after October Break

POLL 1

Calculate the iterated integral of $f(x, y) = x$ on $R = [0,2]\times[0,1]$

$$\int_0^1 \left[\int_0^2 x\,dx\right] dy$$

$a)$ 1

$b)$ 0

$c)$ 2

$$= \int_0^1 \left[\frac{x^2}{2}\right]_0^2 dy = \int_0^1 \left[\frac{2^2}{2} - 0\right] dy$$

$$= \int_0^1 2\,dy = \left[2y\right]_0^1 = 2$$

POLL 2

Let $R = [0,1] \times [0,2]$. Calculate the double integral

$$\int_0^1 \left[\int_0^2 2xye^{xy^2}\,dy\right] dx$$

a) $\frac{e^4}{4}$

b) $\frac{e^4-5}{4}$

c) $e - 1$

d) $\frac{5}{4}$

u-sub

$u = xy^2$

$du = 2xy\,dy$

@ $y=0$ $u=0$

@ $y=2$ $u=4x$

$$= \int_0^1 \int_0^{4x} e^u\,du\,dx$$

$$= \int_0^1 \left[e^u\right]_0^{4x} dx = \int_0^1 e^{4x} - 1\,dx$$

$$= \left[\frac{e^{4x}}{4} - x\right]_0^1 = \frac{e^4}{4} - 1 - \left(\frac{1}{4}\cdot 0\right)$$

$$= \frac{e^4-5}{4}$$

POLL 3

(Spring 2023 Exam 2 #5)

$R = [-1,1] \times [0,5]$ $\quad$ $\text{area}(R) = 2 \cdot 5 = 10$

Find the average value of the function $f(x, y) = x^2y$ over the region R where R is the rectangle with vertices (-1,0), (-1,5), (1,5), and (1,0).

a) $\frac{10}{3}$

b) $\frac{25}{3}$

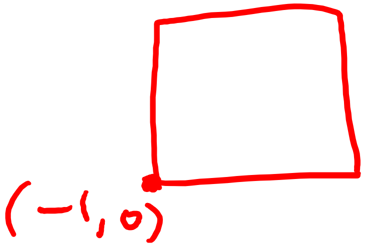
c) $\frac{5}{6}$

d) $\frac{25}{6}$

$$f_{av} = \frac{1}{\text{area}(R)} \iint_R f(x,y)\,dA = \frac{1}{10}\int_{-1}^{1}\int_0^5 x^2y\,dy\,dx$$

$$= \frac{1}{10}\int_{-1}^{1}\left[\frac{x^2y^2}{2}\right]_0^5 dx = \frac{25}{2}\cdot\frac{1}{10}\int_{-1}^{1} x^2\,dx$$

$$= \frac{1}{10}\,\frac{25}{2}\left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{10}\cdot\frac{25}{\cancel{2}}\cdot\frac{\cancel{2}}{3} = \boxed{\frac{5}{6}}$$



MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Double integral
b) Order of operation
c) Volumes of solids
d) Average value
e) None – understood everything today