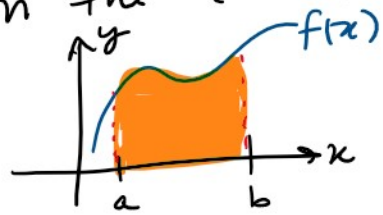
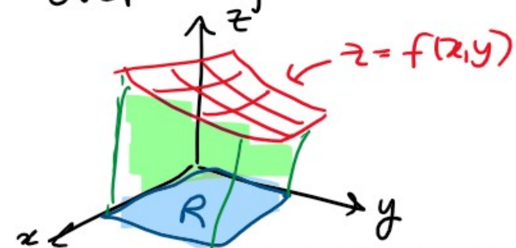
Lesson 18 - 10/4/2023 - 4:30pm

16.1: Double Integrals over Rectangular Regions

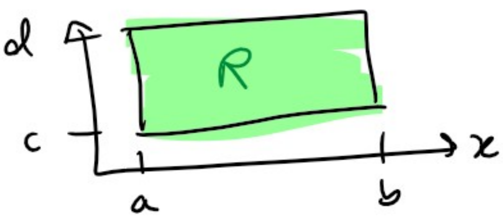
Calc 1/2	Calc 3
$\int_a^b f(x)\,dx$	$\iint_R f(x,y)\,dA$ A – area
definite integral	double integral
area under $f(x)$ on the $[a,b]$	volume under surface $f(x,y)$ over region R
Fundamental Thm of Calculus $\int_a^b f(x)dx = F(b)-F(a)$	?



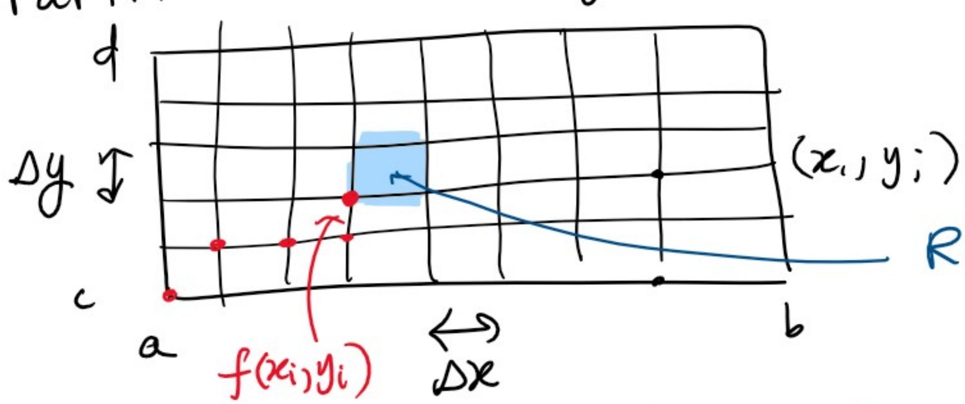


Let's take R = rectangle

$R = [a,b] \times [c,d] = \{(x,y) : a \le x \le b,\ c \le y \le d\}$



Partition R in x and y



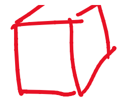
$x_i = a + i\Delta x$

$y_j = c + j\Delta y$

R_{ij} area $DA = \Delta x \Delta y$

rectangular

$- \quad f(x_i, y_i) \quad \Delta x$

Approximate Volume by rectangular 

$$V_{ij} = f(x_i, y_j)\, \Delta A$$

Total Volume

$$V \approx \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\, \Delta A$$

double Riemann sum

$$V = \lim_{(\Delta x, \Delta y) \to 0} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\, \Delta A$$

$$\boxed{V = \iint_R f(x,y)\, dA \qquad \text{double integral}}$$

<u>Def</u>: An <u>iterated</u> <u>integral</u> of $f(x,y)$ over the region $R = [a,b] \times [c,d]$ is:

$$\int_a^b \int_c^d f(x,y)\, dy\, dx = \underbrace{\int_a^b \underbrace{\left[\int_c^d f(x,y)\, dy \right]}_{\text{first integrate wrt } y} dx}_{\text{second integrate wrt } x}$$

order of operations

order of integration

integrate y first

$$\int_c^d \int_a^b f(x,y)\, dx\, dy = \int_c^d \left[\int_a^b f(x,y)\, dx \right] dy$$

<u>Fubini's Thm</u>: Suppose $f(x,y)$ is continuous

... $R = [a,b] \times [c,d]$ then

Fubini's ...

over $R = [a,b] \times [c,d]$ then

$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

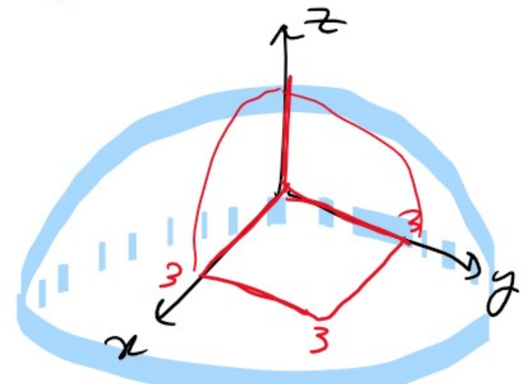
NOTE:

1. We can switch the order of integration
2. one order may be easier to compute

Applications – compute volumes

Ex: Compute the volume V of the solid S that is bounded by the elliptic paraboloid $2x^2 + y^2 + z = 27$, and the planes $x = 3$, $y = 3$ and the coordinate axes $(x=0, y=0, z=0)$

Implicit



Explicit form:

$$z = 27 - 2x^2 - y^2 = f(x,y)$$

$$R = [0,3] \times [0,3]$$

$$V = \iint_R f(x,y)\,dA = \int_0^3 \int_0^3 27 - 2x^2 - y^2 \,dx\,dy$$

Average Values: Calc 1 $f(x)$ over $[a,b]$

$$f_{av} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

Calc 3:

$$f_{av} = \frac{1}{\text{area}(R)} \iint_R f(x,y)\,dA$$