Lesson 19 - 10/11/2023 - 4:30pm

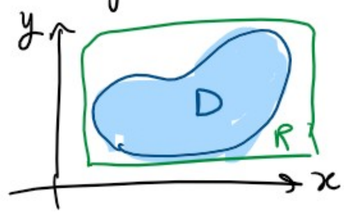
16.2: Double Integrals over General Regions

Last Class: $\iint_R f(x,y)\,dA$ where $R =$ 

Rectangle

Q: How do integrate over general regions?

Suppose $f(x,y)$ is defined on domain D



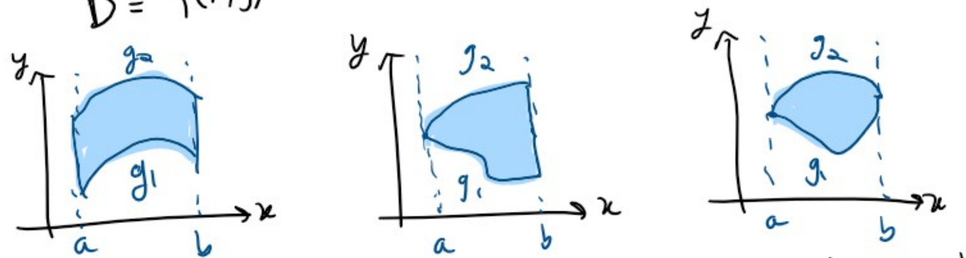
The domain D must be:

1. Bounded (fits inside rectangle R)
2. Bounded by closed simple curves

Def: A region D in the xy-plane is Type I if it lies between two vertical lines and the graphs of two continuous functions $g_1(x)$ and $g_2(x)$

$$D = \{(x,y):\ a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

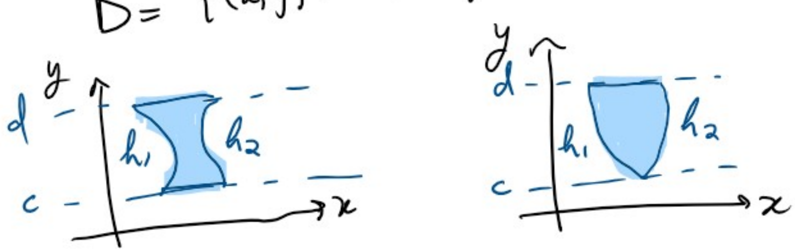
Ex:



Def: D is Type II if it lies between 2 horizontal lines and the graphs of 2 continuous $h_1(y)$ and $h_2(y)$

$$D = \{(x,y):\ c \le y \le d,\ h_1(y) \le x \le h_2(y)\}$$

Ex:



Thm (Fubini's Thm - Strong Form)

For $f(x,y)$ that is continuous on D of Type I, then

<u>Thm</u>

For $f(x,y)$ that is continous on D of Type I, then

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

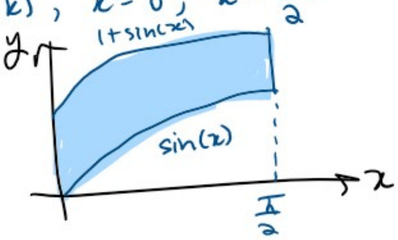
If D is Type II

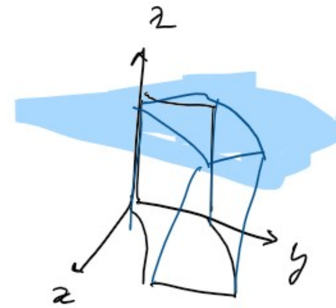
$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

always put constants on the outside integral

<u>Ex:</u> Find the area of the region D bounded by $y=\sin(x)$, $y=1+\sin(x)$, $x=0$, $x=\frac{\pi}{2}$

Type I region





$$V = \iint_D f(x,y)\,dA$$

Let $f(x,y)=1$

$f=z=1$

$V = A \cdot h$
$= A \cdot 1$

$$A = \iint_D 1\,dA$$

$$A = \frac{V}{h}$$

$$= \int_0^{\frac{\pi}{2}} \int_{\sin(x)}^{1+\sin(x)} 1\,dy\,dx = \int_0^{\frac{\pi}{2}} \Big[y\Big]_{\sin(x)}^{1+\sin(x)}\,dx$$

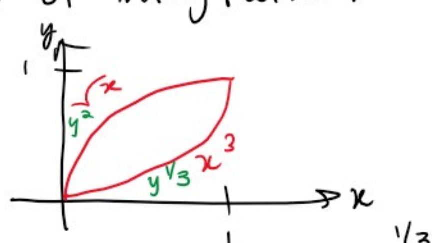
$$= \int_0^{\frac{\pi}{2}} 1 + \cancel{\sin(x)} - \cancel{\sin(x)}\,dx$$

$$= \Big[x\Big]_0^{\frac{\pi}{2}} = \boxed{\frac{\pi}{2} = \text{Area}}$$

<u>NOTE:</u> If D is both Type I and Type II, then we can swith the order of integration

$$\iint_D f(x,y)\,dA = \int_0^1 \int_{x^3}^{\sqrt{x}} f(x,y)\,dy\,dx$$

$$= \int_0^1 \int_{y^2}^{y^{1/3}} f(x,y)\,dx\,dy$$

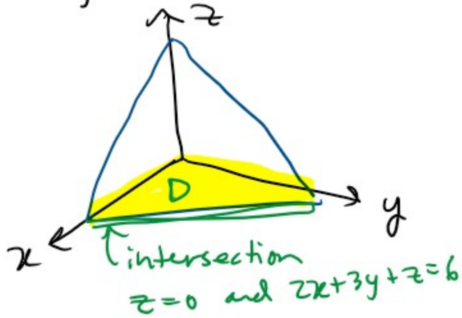


$y = x^3 \longrightarrow x = y^{1/3}$

$y = \sqrt{x} \rightarrow x = y^2$

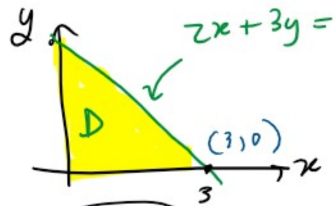
☆ Volumes :

Find the volume of the solid bounded by the planes $x=0$, $y=0$, $z=0$, $2x+3y+z=6$



Volume of tetrahedron

$z = f(x,y) = 6 - 2x - 3y$



$2x + 3y + 0 = 6$

both Type I and Type II

$y = \dfrac{6-2x}{3} = 2 - \frac{2}{3}x = g_1$

$y = g_2(x) = 0$

$$\boxed{V = \iint_D 6 - 2x - 3y \, dA = \int_0^3 \int_0^{2-\frac{2}{3}x} (6-2x-3y)\, dy\, dx}$$