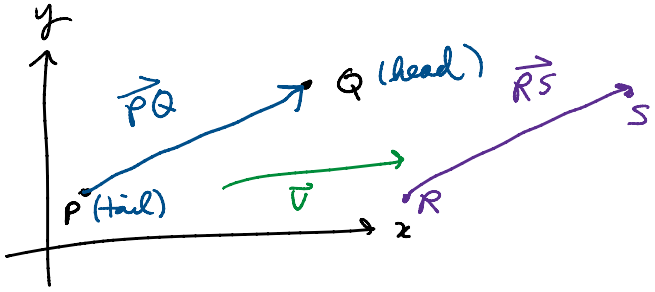
☆ Vectors in the Plane

Def: A vector is a quantity with both a magnitude and a direction

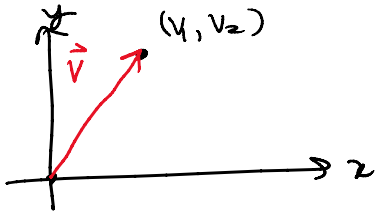


Two vectors are equal if they have the same magnitude and direction $\vec{RS} = \vec{PQ}$

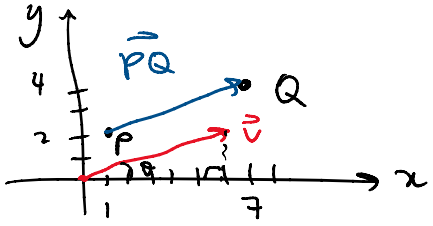
Def: The zero vector $\vec{0}$ has no length and no direction

Def: A scalar is a quantity with magnitude and no direction. Ex: $2, -99, \pi$

Def: A vector $\vec{v}$ with its tail at $(0,0)$ and head at (v_1, v_2) is called a position vector and is written $\langle v_1, v_2 \rangle$



Ex: Let $P(1,2)$ and $Q(7,4)$. Find the position vector $\vec{v}$ equal to $\vec{PQ}$



$\vec{v} = \vec{PQ}$

$= \langle 6, 2 \rangle$

$= \langle 7-1, 4-2 \rangle$

$\boxed{\vec{v} = \langle 6, 2 \rangle}$

magnitude of $\vec{v}$

$|\vec{v}| = \sqrt{6^2 + 2^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

direction $\theta = \tan^{-1}\left(\frac{2}{6}\right) = \tan^{-1}\left(\frac{1}{3}\right) \approx$

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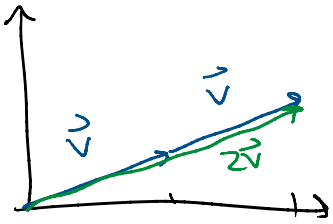
Q: What is $2\vec{v}$?



same direction
2x magnitude

$\vec{v} = \langle 6, 2 \rangle$

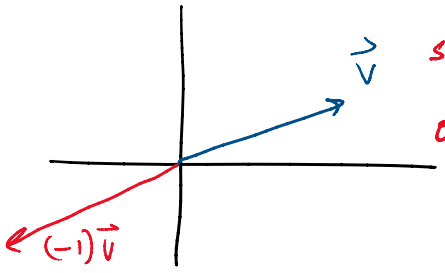
$2\vec{v} = 2\langle 6, 2 \rangle$

$= \langle 2 \cdot 6, 2 \cdot 2 \rangle = \langle 12, 4 \rangle$

Q: What is $(-1)\vec{v}$?

$(-1)\vec{v} = (-1)\langle 6, 2 \rangle$

$= \langle -6, -2 \rangle$



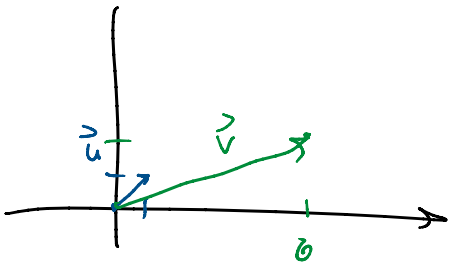
same magnitude
opposite direction

NOTE: Vectors are parallel if they scalar multiples of each other $\vec{v}$, $2\vec{v}$, $-\vec{v}$, $10\vec{v}$

Q: How do you add vectors?

$\vec{u} = \langle 1, 1 \rangle$ and $\vec{v} = \langle 6, 2 \rangle$

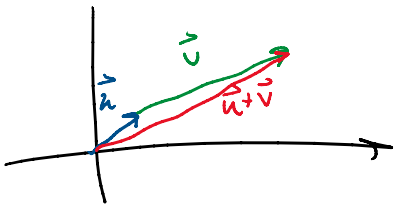
what $\vec{u} + \vec{v}$?



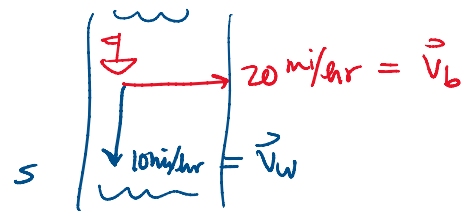
$\vec{u} + \vec{v} = \langle 1, 1 \rangle + \langle 6, 2 \rangle$

$= \langle 1+6, 1+2 \rangle$

$\boxed{\vec{u} + \vec{v} = \langle 7, 3 \rangle}$



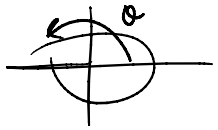
Ex: Water in a river flows south at 10 mi/hr
Boat travels east at 20 mi/hr relative to shore
What is the speed & direction of boat relative to the water?



$$\vec{v}_{rel} = \vec{v}_b + \vec{v}_w = \langle 20\ mi/hr, 0 \rangle + \langle 0, -10\ mi/hr \rangle$$
$$= \langle 20, -10 \rangle\ mi/hr$$

speed $|\vec{v}_{rel}| = \sqrt{(20)^2 + (-10)^2} = \sqrt{500} = 10\sqrt{5}$ mi/hr

direction $\theta = \tan^{-1}\left(\frac{-10}{20}\right) \approx -26.6^\circ$



Def: A **unit vector** is a vector with length 1.

given $\vec{v} = \langle 6, 2 \rangle$

what is the unit vector $\vec{u}$ pointing in same direction?

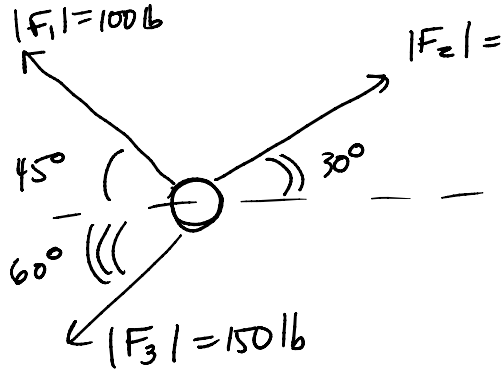
$\vec{u} = k\vec{v} \implies \vec{u} = \frac{\vec{v}}{|\vec{v}|}$

$|\vec{u}| = 1$

$$= \frac{\langle 6, 2 \rangle}{\sqrt{6^2 + 2^2}}$$

$$= \frac{\langle 6, 2 \rangle}{2\sqrt{10}} = \left\langle \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle$$

Net Force:



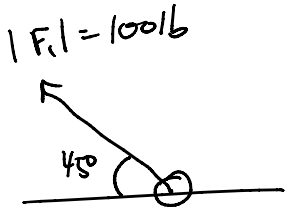
What is the total force on object that

1. Write F_1, F_2, F_3 in position vector form

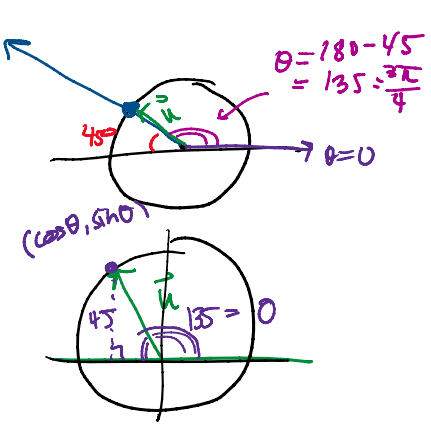
$\vec{F}_1 = \langle -100\cos 45^\circ, 100\sin 45^\circ \rangle$ lb

1. Write F_1, F_2, F_3 in position vector form

2. $F_{net} = F_1 + F_2 + F_3$



What is F_1 in position vector form



What is the unit vector that points in same direction as F_1?

$F_1 = 100lb\ \vec{u}$

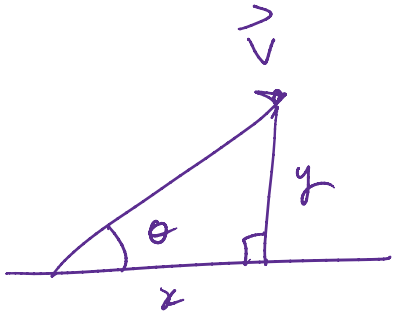
$\vec{u} = \langle x, y \rangle$

$= \langle \cos\left(\frac{3\pi}{4}\right), \sin\left(\frac{3\pi}{4}\right) \rangle$

$= \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$F_1 = 100\ \vec{u} = 100 \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$\boxed{F_1 = \langle -50\sqrt{2}, 50\sqrt{2} \rangle}$



$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$F_2 = \langle a, b \rangle$

$F_3 = \langle c, d \rangle$

$\boxed{F_{net} = F_1 + F_2 + F_3}$