LESSON 20

MA 26100·FALL 2023

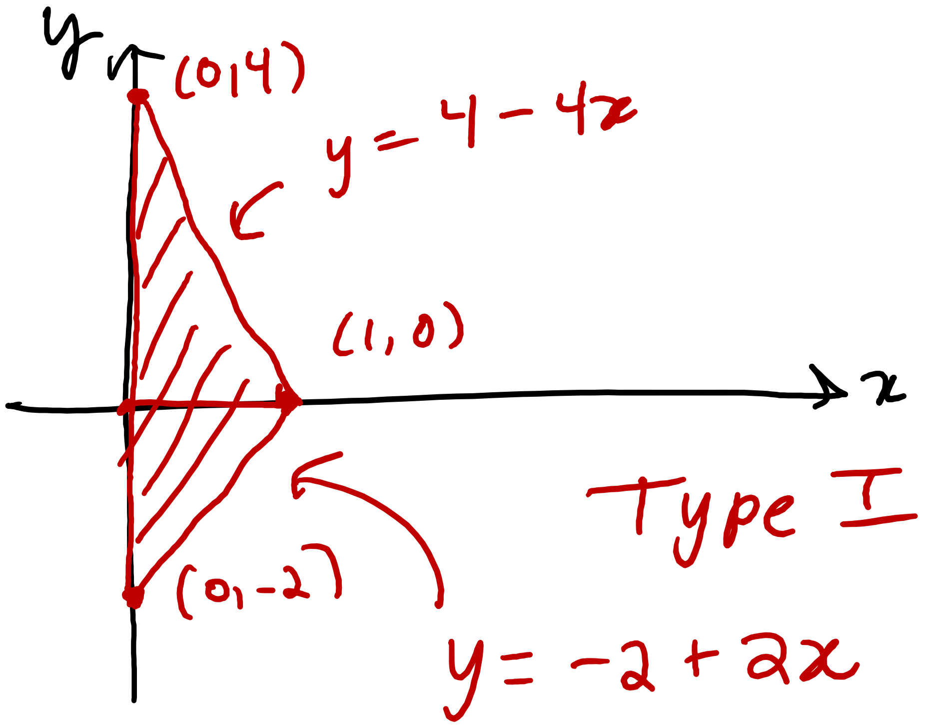
DR. HOOD

WARM UP

(Fall 16 Exam 2 #6)

Let D be the triangle with vertices (0,4), (1,0), and (0,-2). Then

$\iint_D f(x,y)dA$ is:



a) $\int_0^1 \int_{2x-2}^{4-4x} f(x,y)dydx$

b) $\int_0^2 \int_{2+2x}^{4+4x} f(x,y)dydx$

c) $\int_0^4 \int_{2-2x}^{4-4x} f(x,y)dydx$

$0 \le x \le 1$

$2x-2 \le y \le 4-4x$

LESSON 20

ANNOUNCEMENT

- **Correction to Quiz Study Guide:**

19	Double Integrals over General Regions	16.2	6	- Evaluate double integrals over general regions - Change the order of integration	Order of integration	S19E2#2 ~~S19E2#4~~ S19FE#10 F19E2#3 F19FE#9 F18E2#4

POLL 1

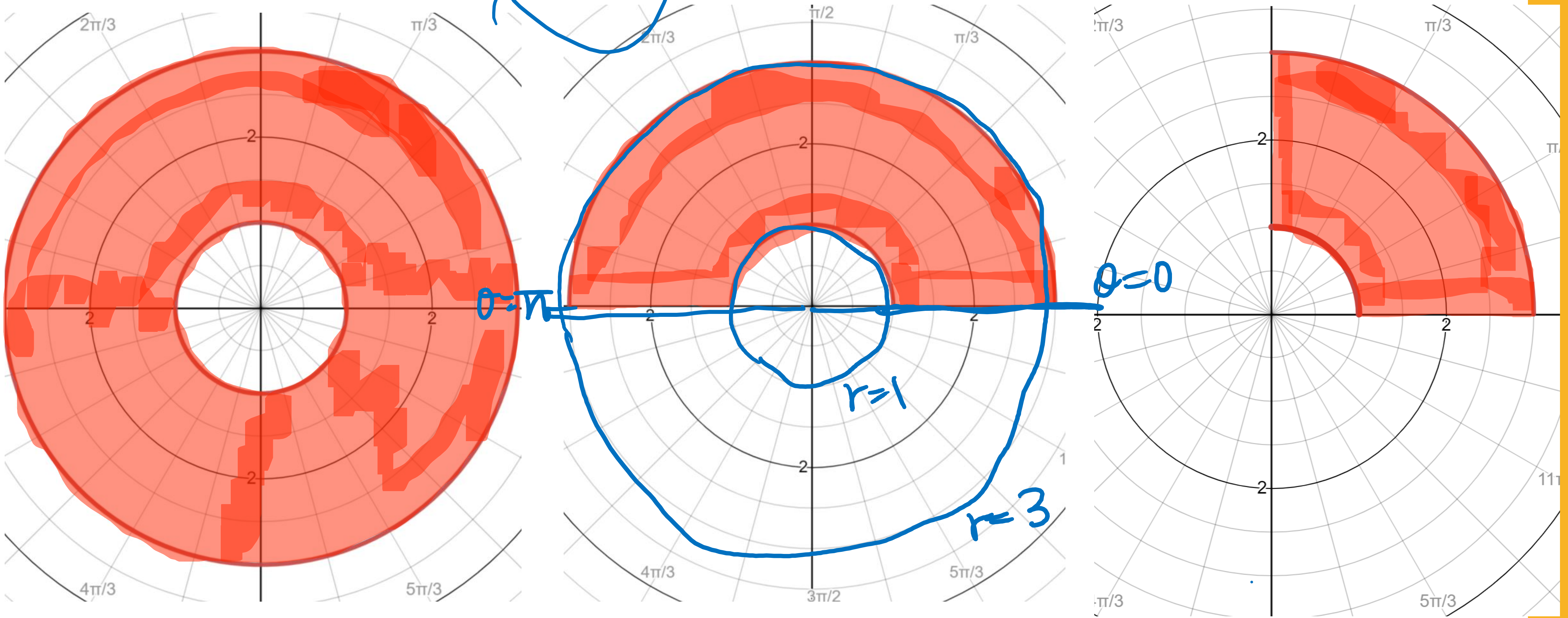
Which of the following regions represents:

$$R = \{(r, \theta) | 1 \leq r \leq 3, 0 \leq \theta \leq \pi\}$$

a) $0 \leq \theta \leq 2\pi$

b)

c) $0 \leq \theta \leq \frac{\pi}{2}$



POLL 2

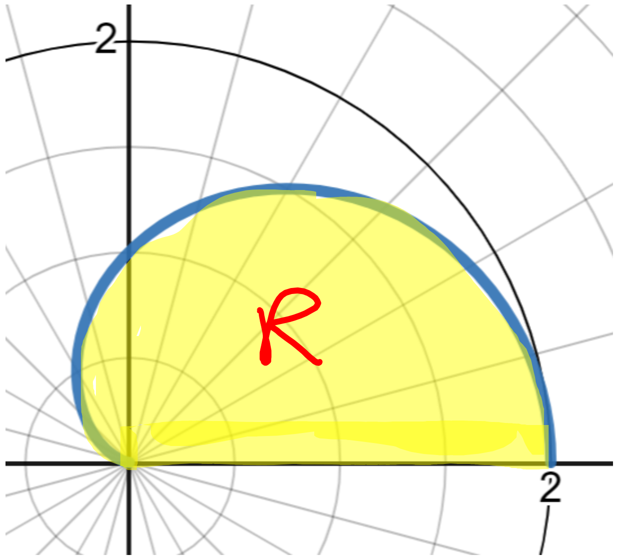
Evaluate $\iint_D r^2 \sin(\theta)\,\overbrace{r\,dr\,d\theta}^{dA}$ for the region D bounded by the polar axis and the upper half of the cardioid $\mathrm{r} = 1 + \cos(\theta)$.

a) 32/5

b) 8

c) 8/5

d) 32



$0 \le \theta \le \pi$

$0 \le r \le 1+\cos\theta$

$$\int_0^{\pi}\int_0^{1+\cos\theta} r^3 \sin\theta \, dr\, d\theta = \int_0^{\pi} \sin\theta \left[\frac{r^4}{4}\right]_0^{1+\cos\theta} d\theta$$

$$= \frac{1}{4}\int_0^{\pi} (1+\cos\theta)^4 \sin\theta \, d\theta$$

$u = 1+\cos\theta$

$du = -\sin\theta\, d\theta$

@ $\theta = 0$ $u = 2$

@ $\theta = \pi$ $u = 0$

$$= -\frac{1}{4}\int_2^0 u^4\, du = -\frac{1}{4}\left[\frac{u^5}{5}\right]_2^0$$

$$= -\frac{1}{4}\left[-\frac{2^5}{5}\right] = \boxed{\frac{8}{5}}$$

POLL 3

(Spring 2015 Exam 2 #6)

$A = \iint_R 1 \cdot r\,dr\,d\theta$

Use integration in polar coordinates to compute the area of the region in the first quadrant inside the circle $(x-1)^2 + y^2 = 1$ and below the line $y = x$. Recall that $2\cos^2(\theta) = 1 + \cos(2\theta)$

$a)\ \frac{\pi}{4} + \frac{1}{2}$

$b)\ \frac{\pi}{2} + \frac{1}{2}$

$c)\ \pi$

$y = x$

$r\sin\theta = r\cos\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

$(r\cos\theta - 1)^2 + (r\sin\theta)^2 = 1$

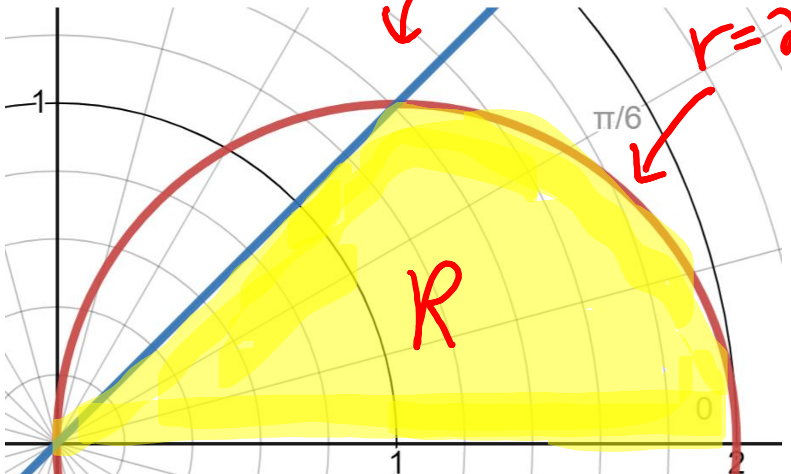
$r^2\cos^2\theta - 2r\cos\theta + \cancel{1} + r^2\sin^2\theta = \cancel{1}$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$\theta = \frac{\pi}{4}$

$r = 2\cos\theta$



$0 \le \theta \le \frac{\pi}{4}$

$0 \le r \le 2\cos\theta$

$$A = \int_0^{\frac{\pi}{4}} \int_0^{2\cos\theta} 1 \cdot r\,dr\,d\theta$$

MUDDIEST POINT

What was the muddiest point from today’s lecture?

a) Integration in polar coordinates
b) General regions in polar
c) Area integrals
d) Setting up the integral
e) None – understood everything today