LESSON 21

MA 26100-FALL 2023

DR. HOOD

WARM UP

(Fall 22 Exam 2 #4)

Let D be the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$. Find $\iint_D e^{x^2+y^2} dA$

$1 \le r \le \sqrt{2}$ $\quad 0 \le \theta \le 2\pi$

a) $\pi e^4 - \pi e$

b) $2\pi e^2 - 2\pi e$

c) $\pi e^2 - \pi e$

d) $2\pi e^2$

e) $6\pi e^4$

$$\int_0^{2\pi} \int_1^{\sqrt{2}} e^{r^2} r \, dr \, d\theta$$

$u = r^2$
$du = 2r\,dr$
@ $r=1$ $u=1$
@ $r=\sqrt{2}$ $u=2$

$$= \frac{1}{2} \int_0^{2\pi} \int_1^2 e^u \, du \, d\theta$$

$$= \frac{1}{2} \int_0^{2\pi} \left[e^u\right]_1^2 d\theta = \frac{1}{2}\left[e^2 - e^1\right] \int_0^{2\pi} d\theta$$

$$= \frac{1}{2}\left[e^2 - e\right] \cdot \left[\theta\right]_0^{2\pi} = \pi e^2 - \pi e$$

LESSON 21

POLL 1

Evaluate the triple integral

$$\iiint_B x^2yz\,dV$$

where: $B = \{(x, y, z) | -1 \leq x \leq 1, 0 \leq y \leq 3, 1 \leq z \leq 3\}$

a) 4

b) 6

c) 12

POLL 2

(Fall 16 Exam 2 #8)

$0 \le x \le y$

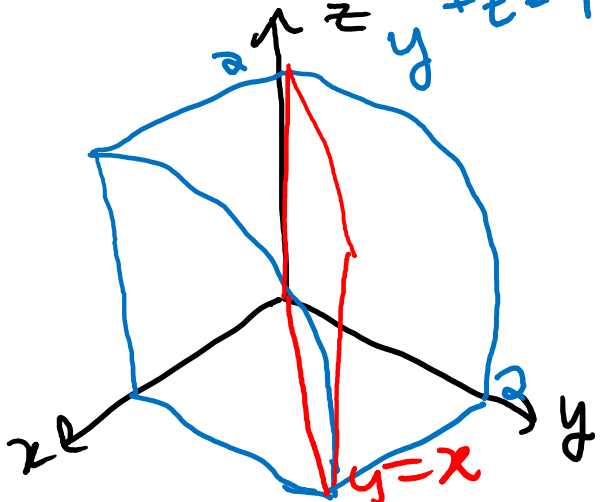
$0 \le z \le \sqrt{4-y^2}$ $\quad 0 \le y \le 2$

Evaluate $\iiint_E z\,dV$ where E is bounded by $y^2 + z^2 = 4$ and the planes $x = 0$, $y = x$, and $z = 0$ in the first octant.

a) 1

b) 2

c) 4



$$\iiint_E z\,dV = \iint_D \left[\int_0^y z\,dx\right] dA = \int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^y z\,dx\,dz\,dy$$

$$= \int_0^2 \int_0^{\sqrt{4-y^2}} z\,[x]_0^y\,dz\,dy = \int_0^2 \int_0^{\sqrt{4-y^2}} yz\,dz\,dy$$

$$= \int_0^2 y\left[\frac{z^2}{2}\right]_0^{\sqrt{4-y^2}} dy = \frac{1}{2}\int_0^2 y(4-y^2)\,dy$$

$$= \frac{1}{2}\int_0^2 4y - y^3\,dy = \frac{1}{2}\left[\frac{4y^2}{2} - \frac{y^4}{4}\right]_0^2 = \frac{1}{2}\left[2\cdot 4 - \frac{2^4}{4}\right]$$

$$= \frac{1}{2}[8-4] = \boxed{2}$$

POLL 3

(Spring 23 Exam 2 #10

Let V be the volume of the solid bounded by the surfaces $z = 6 - x^2 - y^2$ and $z = x^2 + y^2$. If

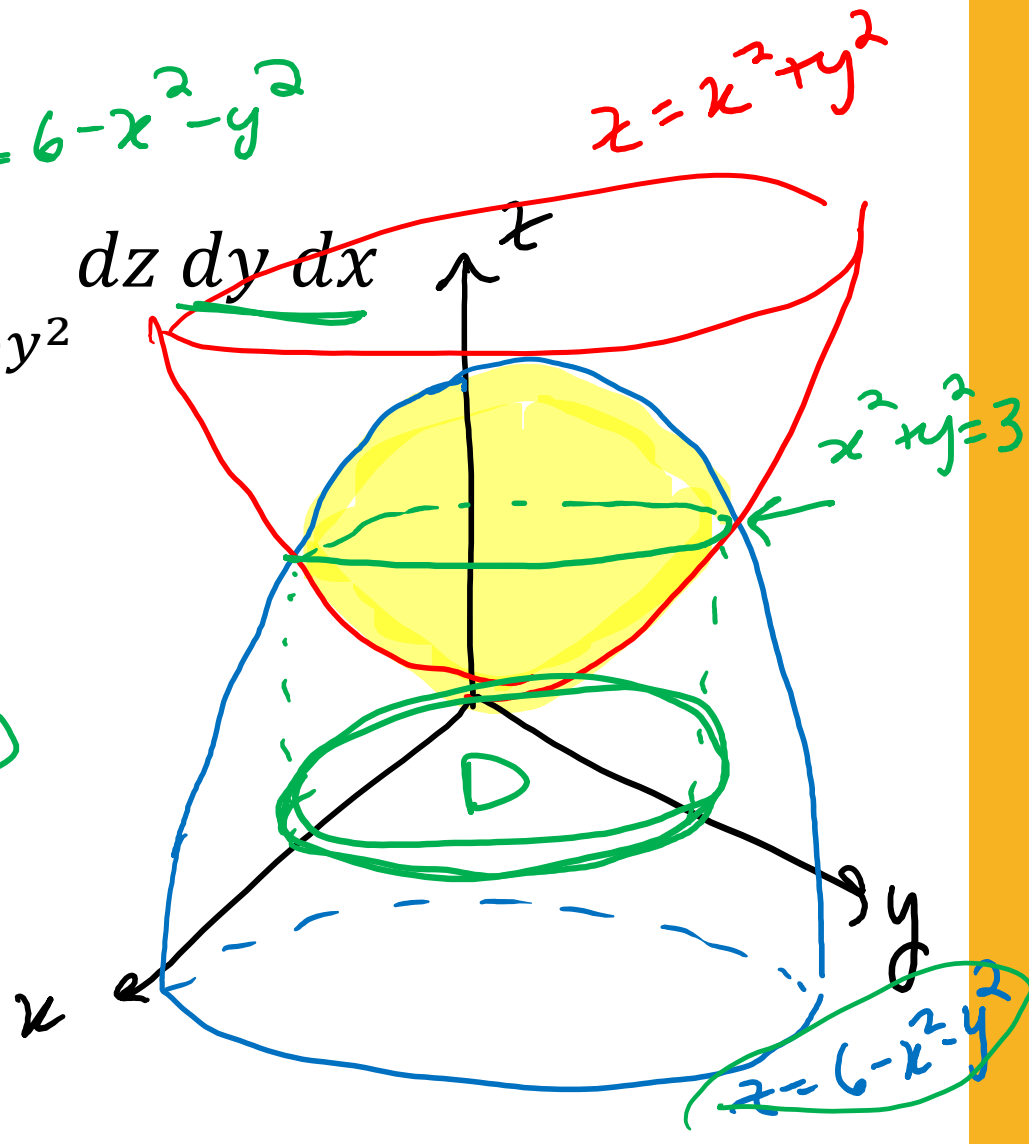
$$V = \int_{-\sqrt{3}}^{\sqrt{3}} \int_{a}^{\sqrt{3-x^2}} \int_{x^2+y^2}^{b} dz\, dy\, dx$$

What are a and b?

a) $a = \sqrt{6 - x^2}, b = \sqrt{3 - x^2 - y^2}$

b) $a = -\sqrt{3 - x^2}, b = 6 - x^2 - y^2$

c) $a = -\sqrt{3 - x^2}, b = 3$



MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Setting up the triple integral
b) Evaluating triple integrals
c) Finding volumes
d) None – understood everything today