Lesson 21 - 10/16/2023 - 4:30pm

16.4: Triple Integrals

	Single	Double	Triple
Domain:	$I=[a,b]$	$R=[a,b]\times[c,d]$	$B=[a,b]\times[c,d]\times[e,f]$
	$\int_a^b f(x)\,dx$	$\iint_R f(x,y)\,dA$	$\iiint_B f(x,y,z)\,dV$
Fubini's Thm		$= \int_a^b\int_c^d f(x,y)\,dy\,dx$ * any order works	$= \int_a^b\int_c^d\int_e^f f(x,y,z)\,dz\,dy\,dx$ * 6 ways to write this integral

Ex: $\iiint_B x^2yz\,dV$ where $B=\{(x,y,z): -1\le x\le 1,\ 0\le y\le 3,\ 1\le z\le 3\}$

$$= \int_{-1}^{1}\int_0^3\int_1^3 x^2yz\,dz\,dy\,dx = \int_0^3\int_1^3\int_{-1}^{1} x^2y^2z\,dx\,dz\,dy$$

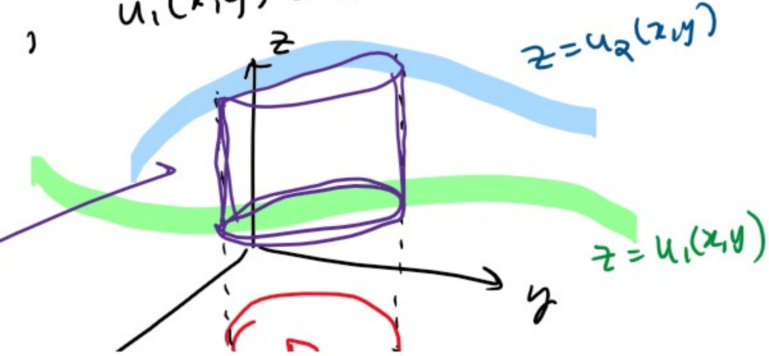
★ General Regions:

Let D be a region in xy-plane. D is in $\mathbb{R}^2$

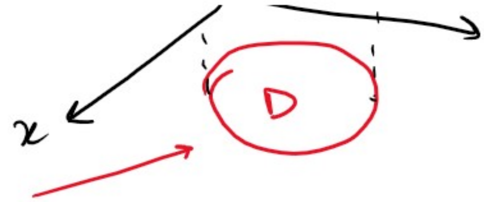
Then E is in $\mathbb{R}^3$ is: "belongs to"

$$E=\{(x,y,z) : (x,y)\in D,\ u_1(x,y)\le z\le u_2(x,y)\}$$

cylinder in z sliced by u_1 and u_2



u_1 and u_2



D is the projection of E onto $z=0$

Then:

$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\right] dA$$

NOTE: Volumes of solids

$$\iint_D u_2(x,y) - u_1(x,y)\ dA = \iiint_E 1 \cdot dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} 1\,dz\right] dA$$

NOTE: We can switch up the variables x, y, and z

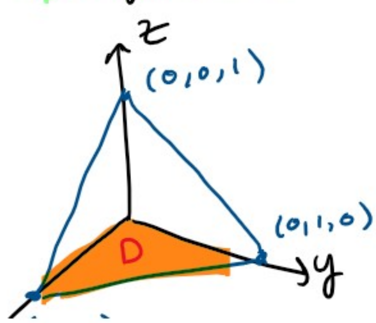
Say D is in yz-plane

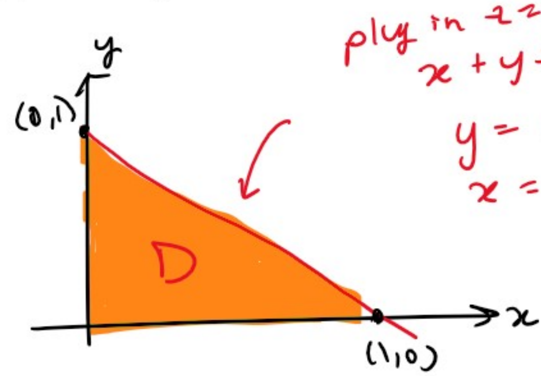
$E = \{(x,y,z) : (y,z) \in D,\ \ v_1(y,z) \le x \le v_2(y,z)\}$

$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{v_1(y,z)}^{v_2(y,z)} f(x,y,z)\,dx\right] dA$$

Example Evaluate $\iiint_E 5x\,dV$ where E is the solid tetrahedron bounded by $x=0$, $y=0$, $z=0$,

$x+y+z=1$



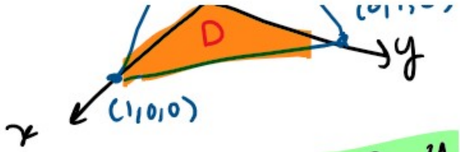


plug in $z=0$ into

$x+y+\cancel{z}^{\,0}=1$

$y = 1-x$

$x = 1-y$



$0 \le z \le 1-x-y$

Type I

$$\begin{cases} 0 \le y \le 1-x \\ 0 \le x \le 1 \end{cases}$$

$$\iiint_E 5x\, dV = \iint_D \left[\int_0^{1-x-y} 5x\, dz \right] dA = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 5x\, dz\, dy\, dx$$

★ **Volume**:

The volume of region E is given by

$$V = \iiint_E 1 \cdot dV$$