LESSON 22

MA 26100-FALL 2023

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WARM UP

(Spring 22 Exam 2 #5)

3. Five of these six triple integrals are over the same region of space: the tetrahedron pictured below with vertices at $(0,0,0)$, $(0,0,1)$, $(1,0,0)$ and $(1,1,0)$. One of these triple integrals is over a different region. Which one is different?

A. $\int_0^1 \int_0^{1-z} \int_y^{1-z} f(x,y,z)\ dx\ dy\ dz$

B. $\int_0^1 \int_0^{1-x} \int_0^{x} f(x,y,z)\ dy\ dz\ dx$

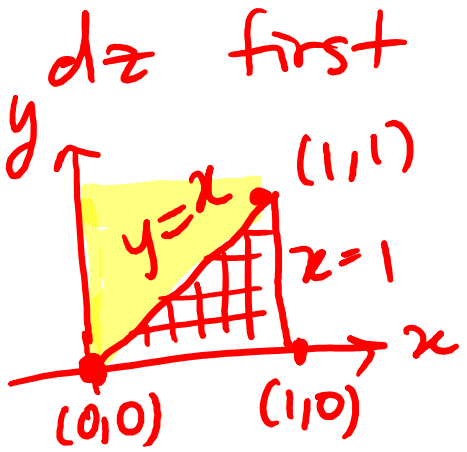
C. $\int_0^1 \int_0^{1-z} \int_0^{x} f(x,y,z)\ dy\ dx\ dz$

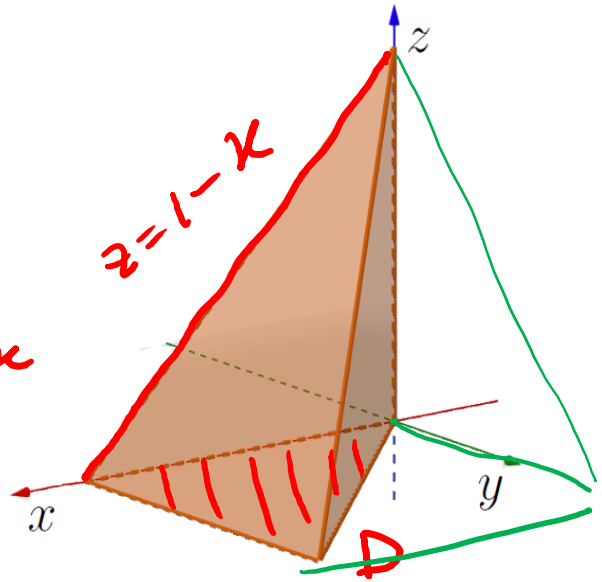
D. $\int_0^1 \int_0^{y} \int_0^{1-x} f(x,y,z)\ dz\ dx\ dy$

E. $\int_0^1 \int_0^{1-y} \int_y^{1-z} f(x,y,z)\ dx\ dz\ dy$

F. $\int_0^1 \int_0^{x} \int_0^{1-x} f(x,y,z)\ dz\ dy\ dx$

dz first





Type I: $\int_0^1 \int_0^{x} \int_0^{1-x} f\ dz\ dy\ dx$ F.

Type II $\int_0^1 \int_y^1 \int_0^{1-x} f\ dz\ dx\ dy$

LESSON 22

POLL 1

Set up the integral to find the volume of the solid cone $z = 3r$ from $z = 0$ to $z = 5$.

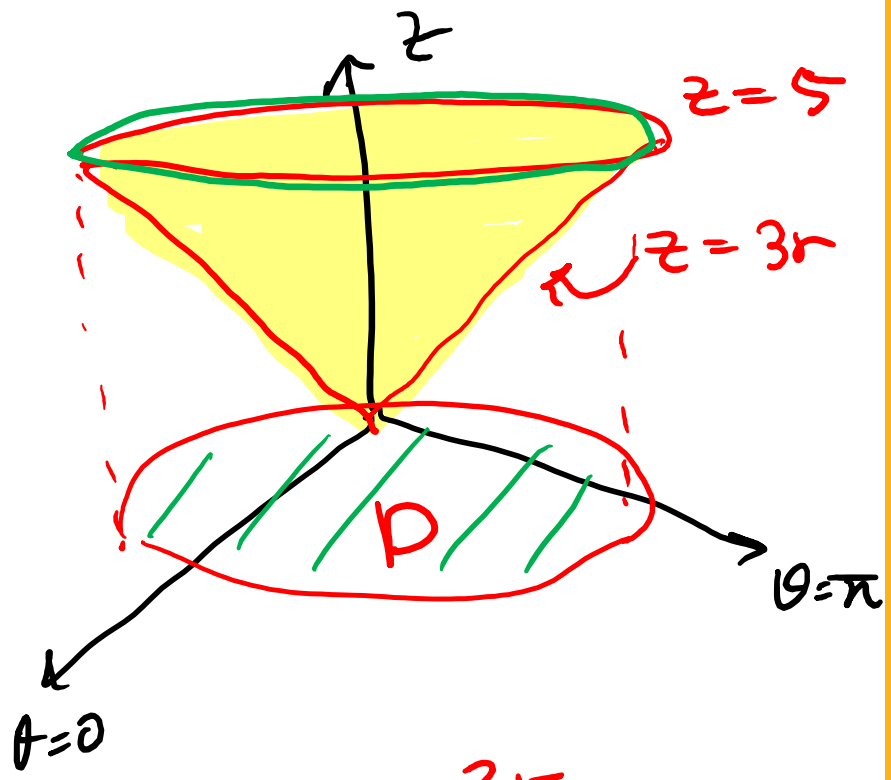
a) $\int_0^{2\pi} \int_0^5 \int_0^{3r} r \, dz \, dr \, d\theta$

b) $\int_0^{2\pi} \int_0^{5/3} \int_{3r}^5 r \, dz \, dr \, d\theta$

$3r \le z \le 5$

$0 \le r \le \frac{5}{3}$

$0 \le \theta \le 2\pi$



$5 = z = 3r$

$\frac{5}{3} = r$

POLL 2

(Spring 23 Exam 2 #11)

$z = \sqrt{x^2+y^2} = r$

$y = \sqrt{100-x^2}$

$x^2 + y^2 = 10^2$

Convert the integral to cylindrical coordinates and evaluate:

$$\int_{x=0}^{10} \int_{0}^{\sqrt{100-x^2}} \int_{0}^{\sqrt{x^2+y^2}} \frac{1}{\sqrt{x^2+y^2}}\, dz\, dy\, dx$$

D

$0 \le \theta \le \frac{\pi}{2} \quad 0 \le r \le 10$

a) 5π

b) 10π

c) 20π

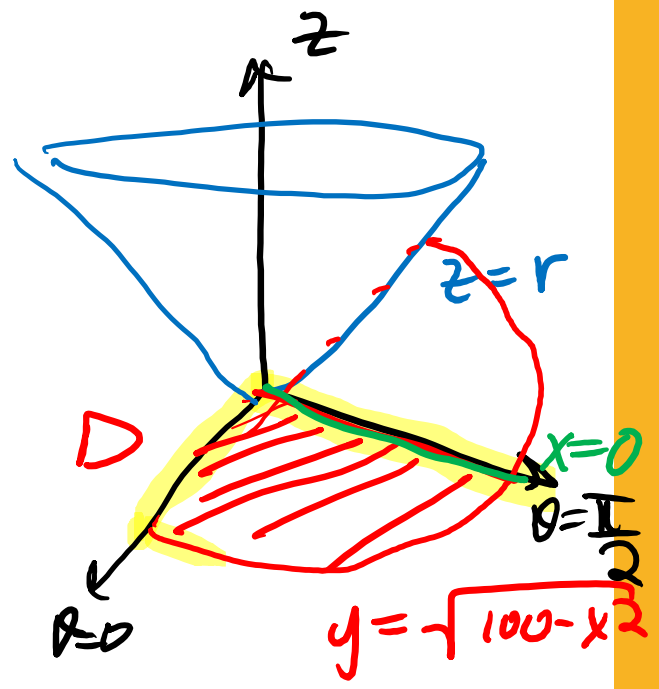
d) 25π

e) 30π

$$\int_0^{\frac{\pi}{2}} \int_0^{10} \int_0^{r} \frac{1}{x} \cdot r\, dz\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{10} r\, dr\, d\theta = \int_0^{\frac{\pi}{2}} \frac{10^2}{2}\, d\theta$$

$$= 50 \left[\theta\right]_0^{\frac{\pi}{2}} = 50 \cdot \frac{\pi}{2} = \boxed{25\pi}$$



POLL 3

(Fall 16 Exam 2 #9

Which of the following integrals represents the volume of the solid enclosed by $z = \sqrt{x^2 + y^2}$ and $x^2 + y^2 + z^2 = 2$. If

a) $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$

b) $\int_0^{2\pi} \int_1^{\sqrt{2}} \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$

c) $\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Converting to cylindrical coordinates
b) Setting up the cylindrical integral
c) Evaluating cylindrical integrals
d) None – understood everything today