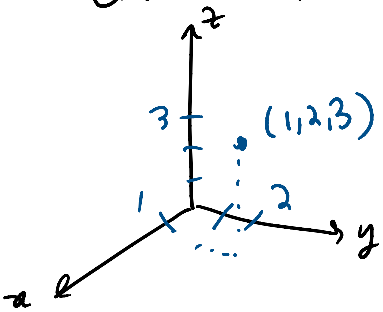
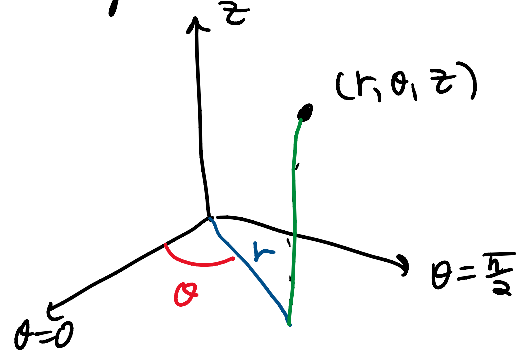
Lesson 22 - 10/18/2023 - 4:30pm

16.5: Triple Integrals in Cylindrical Coordinates

Cartesian



Cylindrical

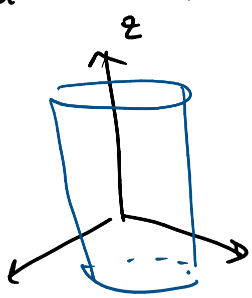


Some common surfaces:

Circular cylinder

Cartesian: $x^2 + y^2 = c^2$

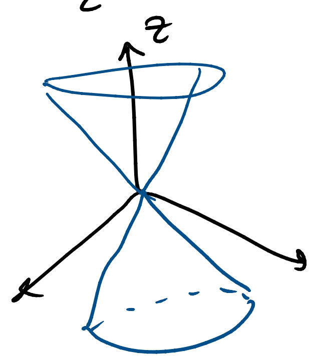
Cylindrical: $r = c$



Circular Cone

$z^2 = c^2(x^2+y^2)$

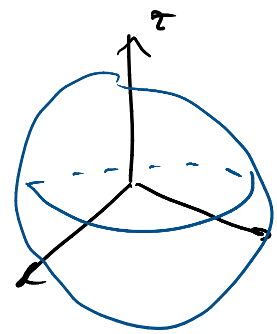
$z = cr$



Sphere

$x^2+y^2+z^2 = c^2$

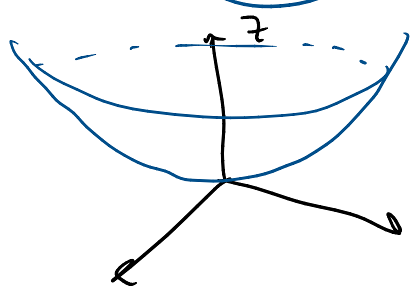
$r^2 + z^2 = c^2$



Paraboloid

$z = c(x^2+y^2)$

$z = cr^2$

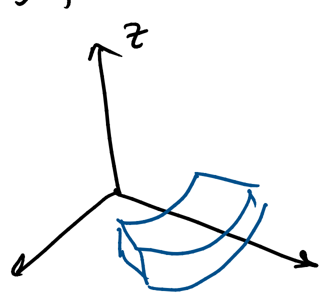


Fubini's Thm.: "cylindrical box"

Let $B = \{(r,\theta,z) : a \le r \le b,\ \alpha \le \theta \le \beta,\ c \le z \le d\}$

$g(x,y,z) = g(r\cos\theta, r\sin\theta, z)$

$= f(r,\theta,z)$



$= f(r, \theta, z)$

then

$$\iiint_B g(x,y,z)\,dV = \int_a^b \int_\alpha^\beta \int_c^d f(r,\theta,z)\,dz\,r\,dr\,d\theta$$

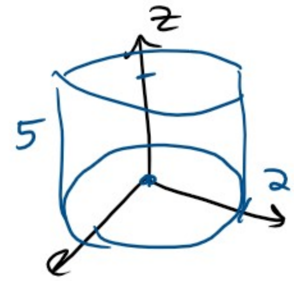
<u>Example</u>: Integrate $\iiint_B (zr\sin\theta)\,r\,dr\,d\theta\,dz$

over the solid cylinder of radius 2, height 5, whose base is centered at (0,0,0)

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$0 \le z \le 5$

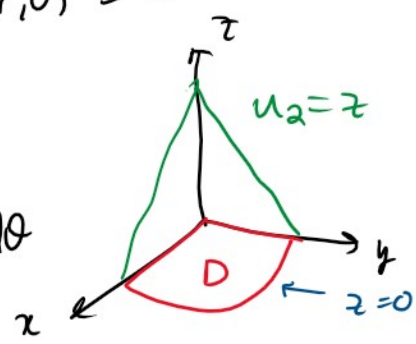


$$\int_0^5 \int_0^{2\pi} \int_0^2 (zr\sin\theta)\,r\,dr\,d\theta\,dz$$

☆ <u>General Regions</u>:

Let D be in the xy-plane and E is a solid in $\mathbb{R}^3$

$$E = \{(r,\theta,z) : (r,\theta) \in D,\ u_1(r,\theta) \le z \le u_2(r,\theta)\}$$

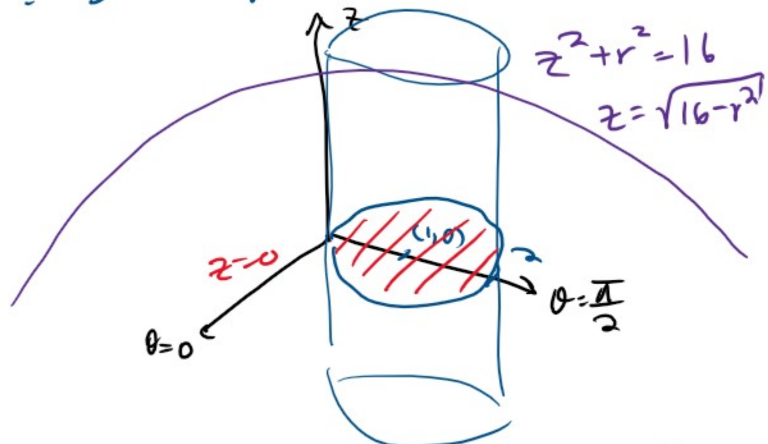


Then

$$\iiint_E f(r,\theta,z)\,dV = \iint_D \left[\int_{u_1(r,\theta)}^{u_2(r,\theta)} f(r,\theta,z)\,dz\right] r\,dr\,d\theta$$

Example: Consider region E inside the circular ... below by $z=0$

Example: Consider region E inside the circular cylinder $r = 2\sin\theta$, bounded below by $z = 0$ and above $z^2 + r^2 = 16$. Set up the integral



in polar:
$r = 2a\cos\theta + 2b\sin\theta$
is a circle with
center (a, b)
radius $\sqrt{a^2+b^2}$

$r = 2\sin\theta \rightarrow a = 0, \; b = 1$
center $(0, 1)$
radius $\sqrt{0^2+1^2} = 1$

$0 \le z \le \sqrt{16-r^2}$
$0 \le r \le 2\sin\theta$
$0 \le \theta \le \cancel{2\pi}\ \pi$

$$\iiint_E f(r,\theta,z)\,dV = \int_0^{\cancel{2\pi}\ \pi} \int_0^{2\sin\theta} \int_0^{\sqrt{16-r^2}} f(r,\theta,z)\, r\, dz\, dr\, d\theta$$