LESSON 23

MA 26100-FALL 2023

DR. HOOD

WARM UP

(Spring 22 Exam 2 #8)

8. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{-1}^{1}(x^2+y^2)^{3/2}\,dz\,dx\,dy$

A. $\frac{2\pi}{7}$

B. $\frac{2\pi}{5}$

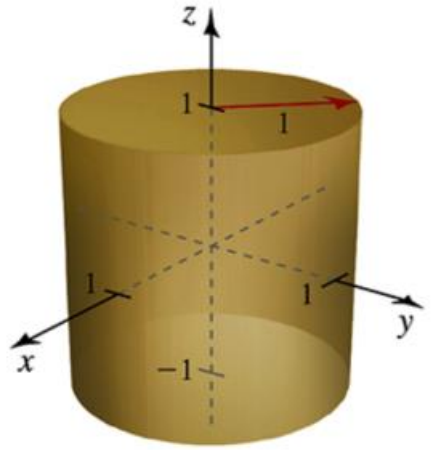
C. $\frac{4\pi}{5}$

D. $\frac{4\pi}{7}$

E. $\frac{\pi}{2}$

F. π

1 → $V=\pi(r)^2h$



$-1\le z\le 1$

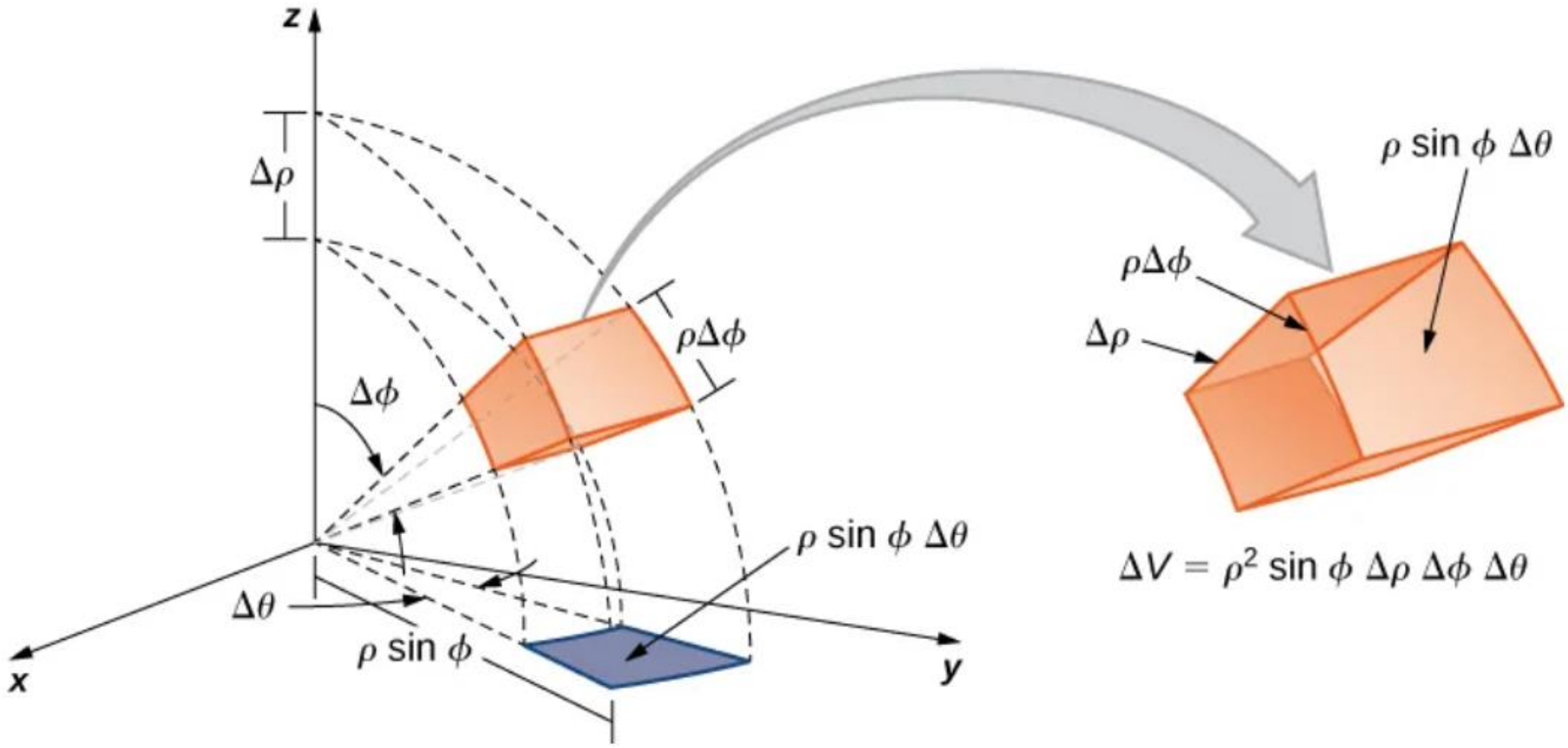
$0\le r\le 1$ $\quad 0\le\theta\le 2\pi$

$$\int_0^{2\pi}\int_0^1\int_{-1}^{1} r^3\, r\,dz\,dr\,d\theta = 2\int_0^{2\pi}\int_0^1 r^4\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\left[\frac{r^5}{5}\right]_0^1 d\theta = \frac{2}{5}\cdot 2\pi = \boxed{\frac{4\pi}{5}}$$

LESSON 23

SPHERICAL BOX



POLL 1

Use integration in spherical coordinates to find the volume of a sphere of radius a.

a) $4\pi a^2$

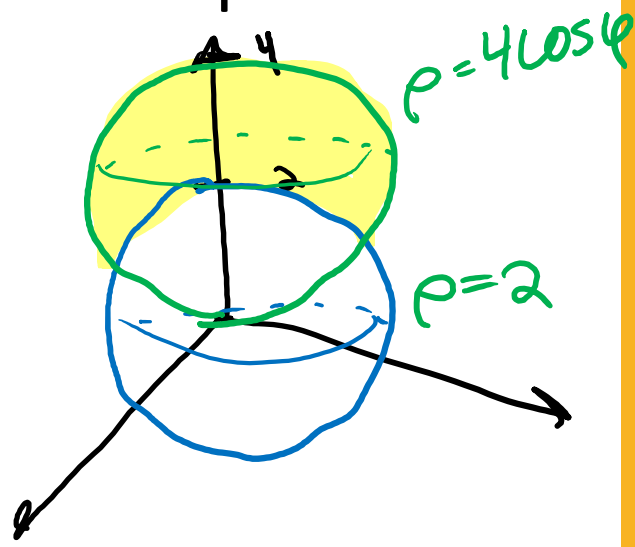
b) $\frac{4}{3}\pi a^3$

c) $\frac{2}{3}\pi a^3$

POLL 2

(Spring 23 Exam 2 #11)

Which of the following represents the volume of the solid inside a sphere of radius 2 centered at (0, 0, 2) and outside the sphere of radius 2 centered at (0,0,0)?



a) $\int_0^{2\pi} \int_0^{\pi/3} \int_2^{4\cos\varphi} \rho^2 \sin\varphi \; d\rho \, d\varphi \, d\theta$

b) $\int_0^{2\pi} \int_0^{\pi/3} \int_0^{4\cos\varphi} \rho^2 \sin\varphi \; d\rho \, d\varphi \, d\theta$

c) $\int_0^{2\pi} \int_0^{\pi/3} \int_{4\cos\varphi}^{2} \rho^2 \sin\varphi \; d\rho \, d\varphi \, d\theta$

POLL 3

(Spring 22 Exam 2 #6)

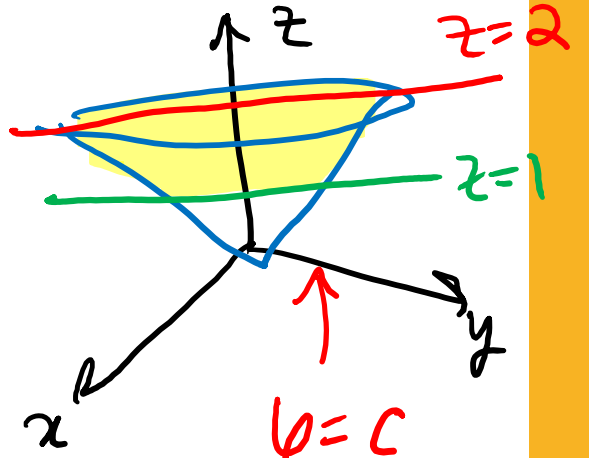
$1 = z = \rho\cos\varphi$

$\sec\varphi = \rho$

$2 = z = \rho\cos\varphi$

$\rho = 2\sec\varphi$

Choose the triple integral in spherical coordinates that represents the volume of the solid bounded by the cone $z^2 = x^2 + y^2$ and lying between planes $z = 1$ and $z = 2$.



$0 \le \varphi \le c$

$\sec\varphi \le \rho \le 2\sec\varphi$

$d\varphi\, d\rho$

a) $\int_0^{2\pi}\int_0^{2\sqrt{2}}\int_0^{\pi/4} \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$

b) $\int_0^{2\pi}\int_0^{\pi/4}\int_{\sec\varphi}^{2\sec\varphi} \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$

c) $\int_0^{2\pi}\int_0^{2}\int_0^{\pi/4} \rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$

$d\varphi\, d\rho$

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Converting to spherical coordinates
b) Setting up the spherical integral
c) Evaluating spherical integrals
d) None – understood everything today