LESSON 24

MA 26100·FALL 2023

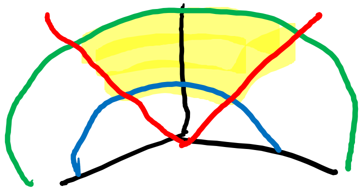
DR. HOOD

WARM UP

(Fall 16 Exam 2 #10)

$1 \le \rho \le 2$ $\quad 0 \le \theta \le 2\pi$

$0 \le \phi \le \frac{\pi}{4}$



Find the volume of the solid enclosed by $x^2 + y^2 + z^2 = 1$, $\Rightarrow \rho = 1$

$x^2 + y^2 + z^2 = 4$, and $z = \sqrt{x^2 + y^2}$. $\rightarrow \phi = c = \frac{\pi}{4}$

$\Downarrow$

$\rho = 2$

a) $\frac{14\pi}{3}\left(1 - \frac{\sqrt{2}}{2}\right)$

b) $\frac{14\pi}{3}\left(1 + \frac{\sqrt{2}}{2}\right)$

c) $3\pi\left(1 - \frac{\sqrt{2}}{2}\right)$

$$\int_0^{2\pi}\int_1^2\int_0^{\frac{\pi}{4}} \rho^2 \sin\phi \, d\phi \, d\rho \, d\theta$$

$$= \int_0^{2\pi}\int_1^2 \left[-\cos\phi\right]_0^{\frac{\pi}{4}} \rho^2 \, d\rho \, d\theta = \left[-\frac{\sqrt{2}}{2} + 1\right]\int_0^{2\pi}\int_1^2 \rho^2 \, d\rho \, d\theta$$

$$= \left[1 - \frac{\sqrt{2}}{2}\right]\int_0^{2\pi}\left[\frac{\rho^3}{3}\right]_1^2 d\theta = \left[1 - \frac{\sqrt{2}}{2}\right]\left[\frac{8}{3} - \frac{1}{3}\right]2\pi$$

$$= \frac{14\pi}{3}\left[1 - \frac{\sqrt{2}}{2}\right]$$

LESSON 24

ANNOUNCEMENTS

- Dr. Hood must leave promptly after the 4:30pm class to substitute for another class

POLL 1

Given a see-saw with mass m_1 at x_1 and mass m_2 at x_2, find the center of mass $\bar{x}$. (Hint: set the moments equal).

$x_1 < \bar{x} < x_2$

a) $\bar{x} = \frac{x_1+x_2}{2}$

b) $\bar{x} = \frac{m_1x_1+m_2x_2}{2m_1}$

c) $\bar{x} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$

$$m_1|x_1-\bar{x}| = m_2|x_2-\bar{x}|$$

$$m_1(\bar{x}-x_1) = m_2(x_2-\bar{x})$$

$$m_1\bar{x} + m_2\bar{x} = m_1x_1 + m_2x_2$$

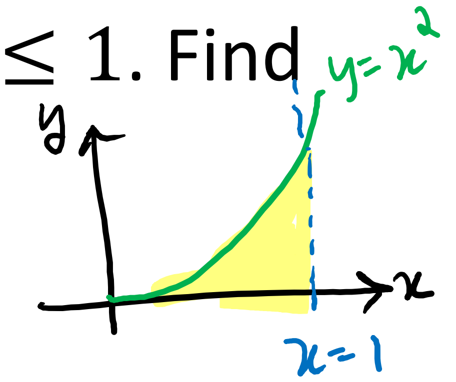
$$\bar{x} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$$

total mass moment

total mass

POLL 2

Let R be the region under the curve $y = x^2$ for $0 \le x \le 1$. Find the moment M_x for the density $\rho(x, y) = x + y$.



$$M_x = \iint_D y\,\rho(x,y)\,dA$$

$$= \int_0^1 \int_0^{x^2} y(x+y)\,dy\,dx = \int_0^1 \left[\frac{y^2}{2}x + \frac{y^3}{3}\right]_0^{x^2} dx$$

$$= \int_0^1 \frac{x^5}{2} + \frac{x^6}{3}\,dx = \left[\frac{x^6}{12} + \frac{x^7}{3\cdot 7}\right]_0^1 = \frac{1}{12} + \frac{1}{21} = \frac{11}{84}$$

a) $\frac{7}{20} = m$

b) $\frac{17}{60} = M_y$

c) $\frac{11}{84} = M_x$

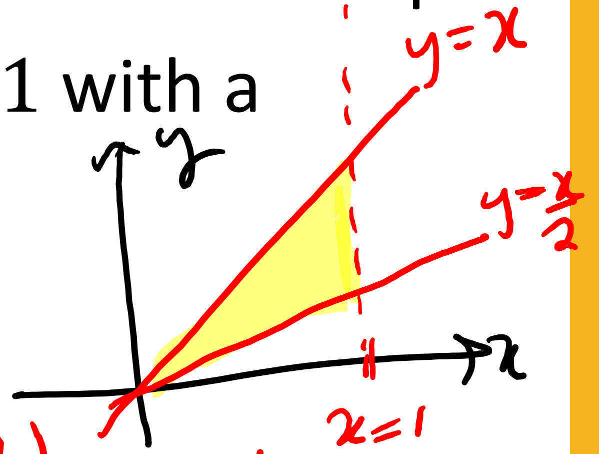
$$\bar{x} = \frac{M_y}{m} = \frac{17/60}{7/20} = \frac{17}{21}$$

$$\bar{y} = \frac{M_x}{m} = \frac{11/84}{7/20} = \frac{55}{147}$$

POLL 3

(Spring 23 Exam 2 #12)

Find the x-coordinate of the center of mass of a plate in the shape of the region bounded by $y = x$, $y = \frac{1}{2}x$, and $x = 1$ with a density $\rho(x, y) = 2x$.



a) $\frac{4}{5}$

b) $\frac{5}{6}$

c) $\frac{2}{3}$

d) $\frac{3}{4}$

e) $\frac{6}{7}$

$$m = \int_0^1 \int_{\frac{x}{2}}^{x} 2x \, dy \, dx$$

$$= \int_0^1 \left[2xy\right]_{\frac{x}{2}}^{x} dx = \int_0^1 x^2 \, dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\bar{x} = \frac{1}{m} \int_0^1 \int_{\frac{x}{2}}^{x} x \cdot 2x \, dy \, dx = 3 \int_0^1 2x^2 \left[y\right]_{\frac{x}{2}}^{x} dx$$

$$= 3 \int_0^1 x^3 \, dx = 3 \left[\frac{x^4}{4}\right]_0^1 = \boxed{\frac{3}{4}}$$

POLL 4

Find the z-coordinate of the center of the solid elliptic parabola $z = x^2 + y^2$ between $z = 0$ and $z = 4$ with constant density $\rho(x, y) = 1$.

$a)\ \frac{7}{8}$

$b)\ \frac{21}{8}$

$c)\ 2$

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Center of mass equations
b) Moments
c) Solving for center of mass
d) None – understood everything today