LESSON 27

MA 26100-FALL 2023

DR. HOOD

WARM UP

(Spring 23 Final Exam #13)

Compute the line integral $\int_C (2x + y)\, ds$ where C is the line segment from $(0,0)$ to $(6,8)$.

a) 80

b) 120

c) 140

d) 100

e) 160

direction vector $\vec{v} = \langle 6, 8 \rangle$

$\vec{r}(t) = \langle 6t, 8t \rangle \quad 0 \le t \le 1$

$|\vec{r}'(t)| = |\vec{v}| = \sqrt{6^2 + 8^2} = 2\sqrt{3^2 + 4^2} = 2 \cdot 5 = 10$

$$\int_0^1 (2(6t) + 8t) \cdot 10 \, dt = 200 \int_0^1 t \, dt$$

$$= 200 \left[\frac{t^2}{2} \right]_0^1 = 100$$

LESSON 27

POLL 1

Find the value of $\int_C \vec{F} \cdot \vec{T} ds$ where C is the semicircle parameterized by $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$ for $0 \leq t \leq \pi$, and $\vec{F}(x, y) = \langle -y, x \rangle$

$\vec{r}\,'(t) = \langle -\sin(t), \cos(t) \rangle$

a) 0

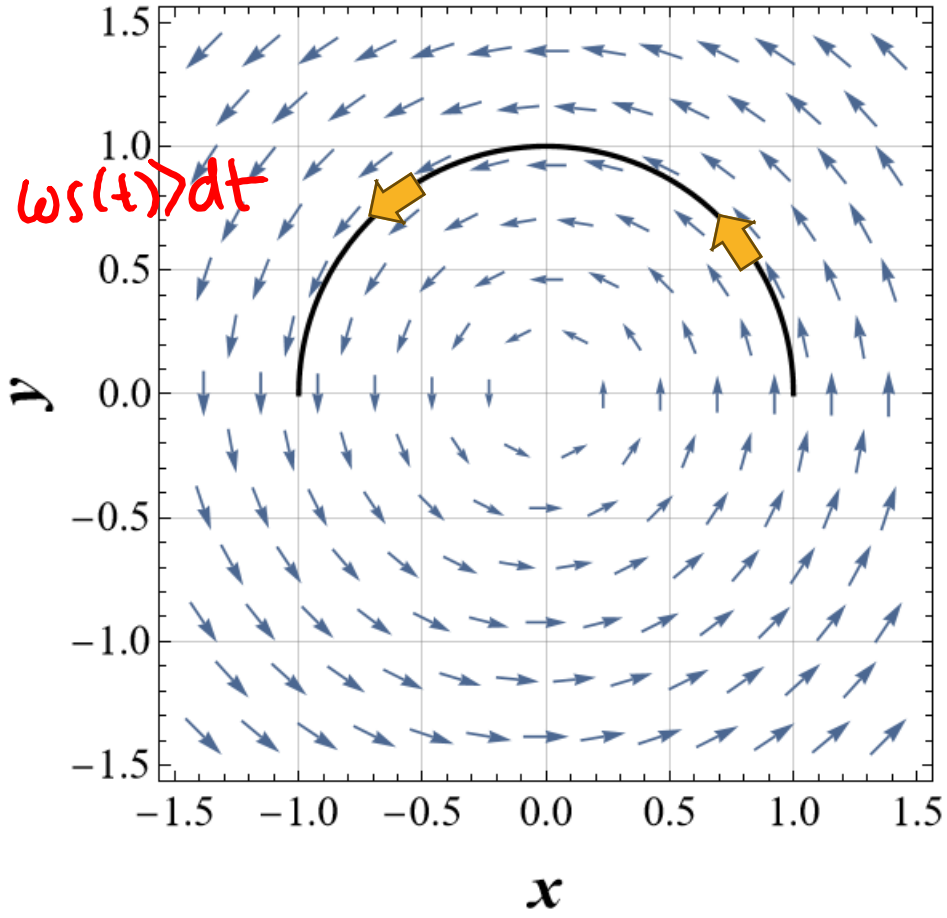
b) π

c) 2π

$$\int_0^{\pi} \langle -\sin(t), \cos(t) \rangle \cdot \langle -\sin(t), \cos(t) \rangle \, dt$$

$$= \int_0^{\pi} \sin^2(t) + \cos^2(t) \, dt$$

$$= \int_0^{\pi} 1 \cdot dt = \pi$$



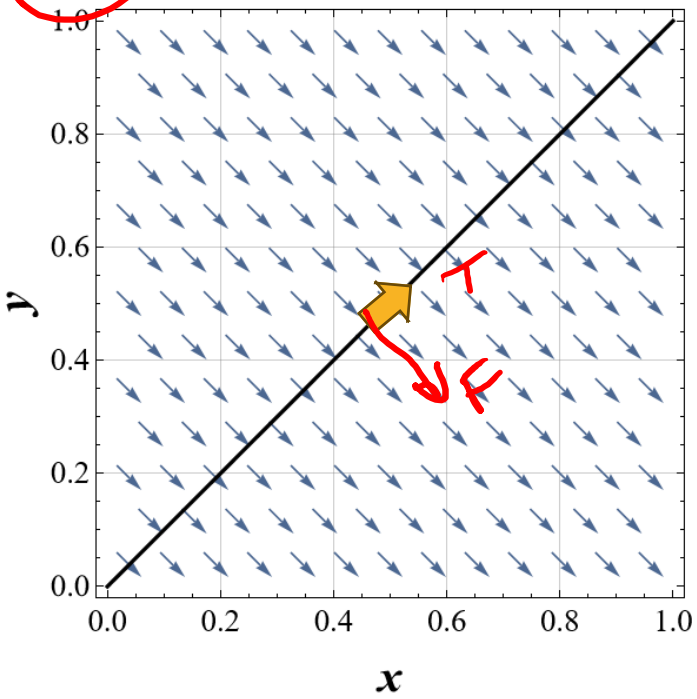
POLL 2

Which of the following vector fields $\vec{F}(x, y)$ results in the **least** amount of work along the curve C parameterized by $\vec{r}(t) = \langle t, t \rangle$ for $0 \leq t \leq 1$?

$|\vec{F} \cdot \vec{T}| = |\vec{F}||\vec{T}| \cos\theta = |\vec{F}|$

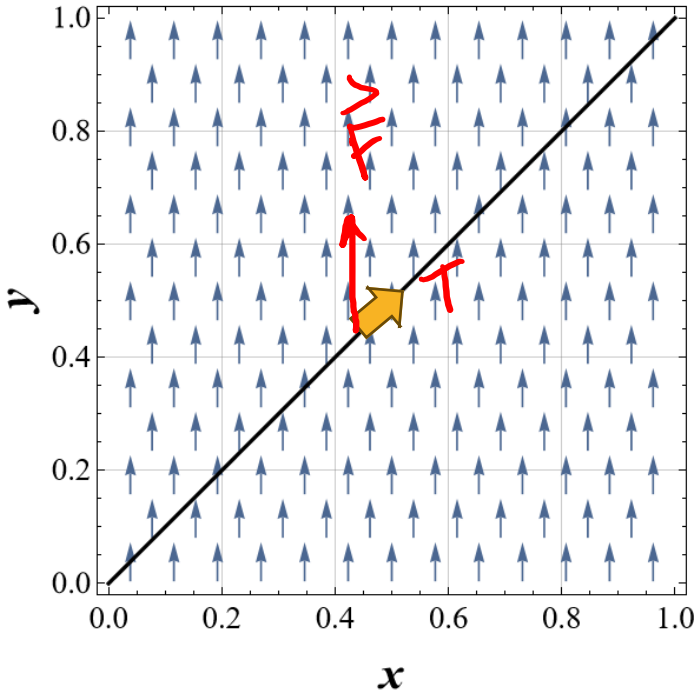
$\vec{F} \parallel \vec{T}$

a) $\vec{F} \perp \vec{T}$



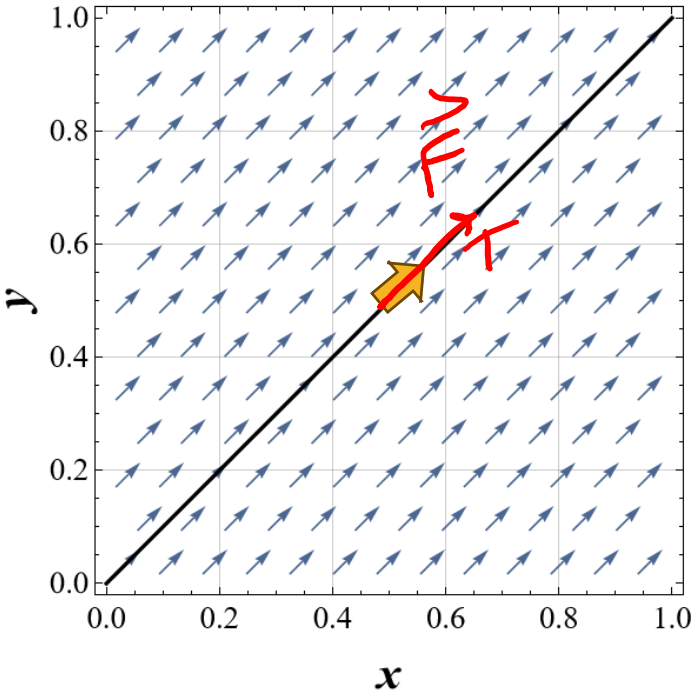
$\vec{F} \cdot \vec{T} = 0$

b)



$|\vec{F} \cdot \vec{T}| = |\vec{F}| \cos\theta$

c)



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POLL 3

Calculate the flux of $\vec{F} = \langle 2x, 2y \rangle$ across the unit circle oriented counterclockwise.

a) 0

b) 2π

c) 4π

d) 8π

$\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$

$\vec{r}\,'(t) = \langle -\sin(t), \cos(t) \rangle$

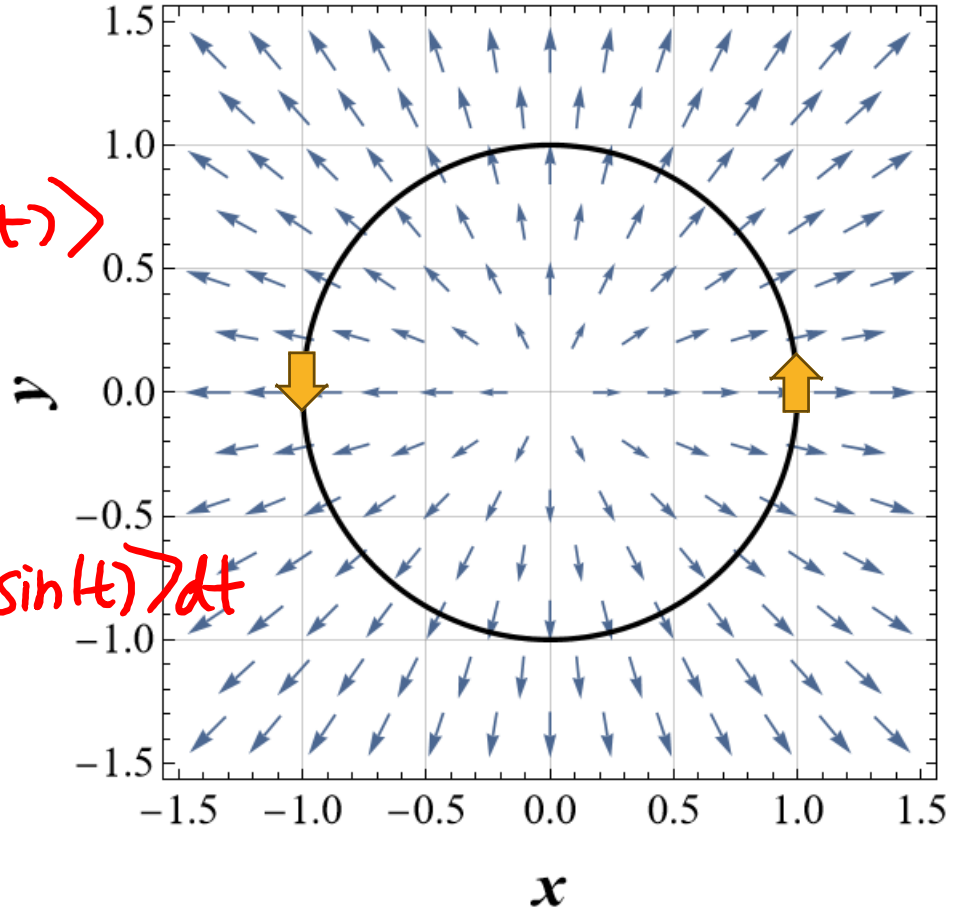
$\vec{n} = \langle y, -x \rangle = \langle \cos(t), \sin(t) \rangle$

$$\int_C \vec{F} \cdot \vec{N}\, ds$$

$$= \int_0^{2\pi} \langle 2\cos(t), 2\sin(t) \rangle \cdot \langle \cos(t), \sin(t) \rangle\, dt$$

$$= \int_0^{2\pi} 2\cos^2(t) + 2\sin^2(t)\, dt$$

$$\int_0^{2\pi} 2\, dt = 2 \cdot 2\pi = 4\pi$$



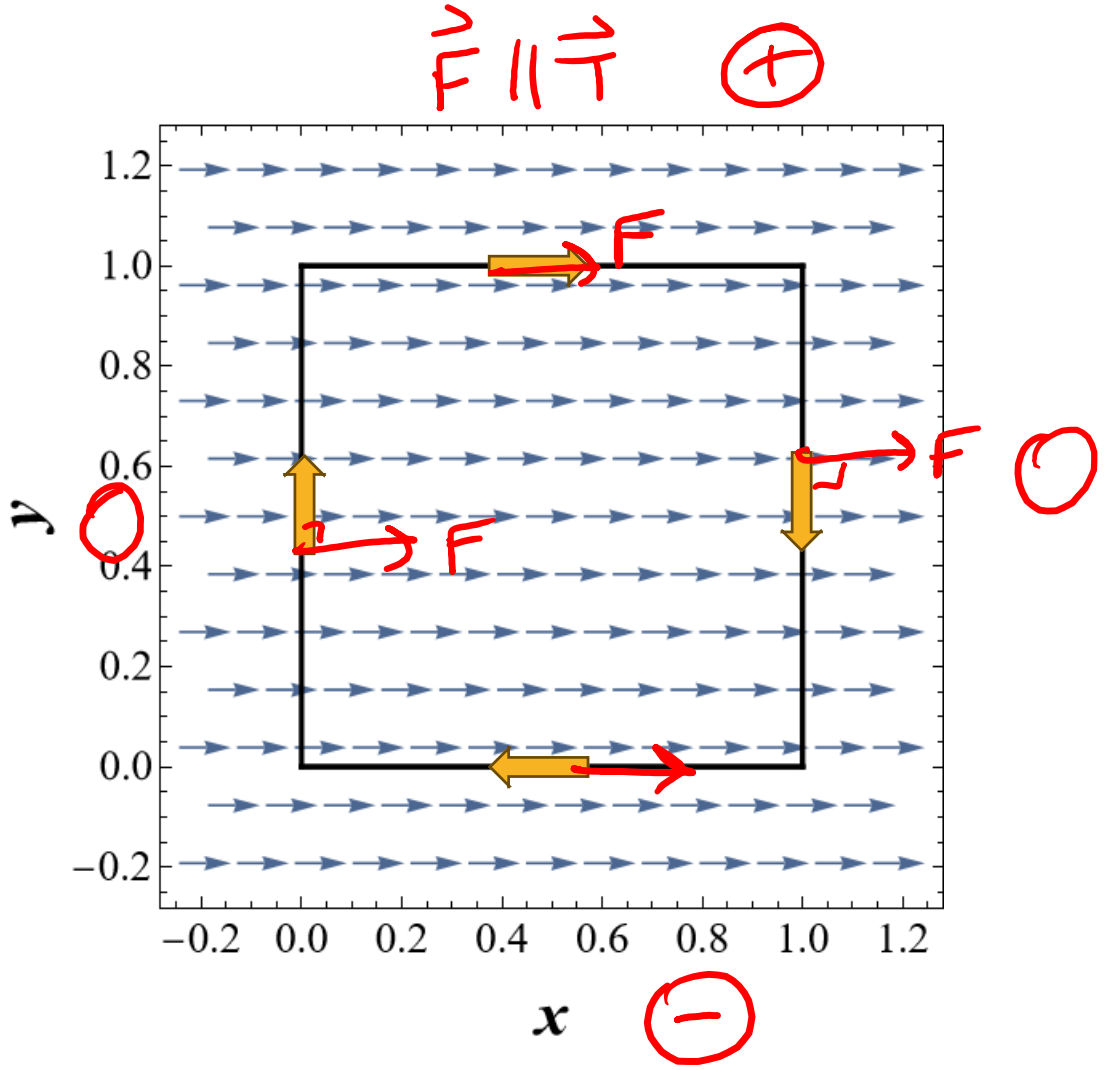
POLL 4

Estimate the circulation of the vector field in the where curve C is the square with corners at (0,0), (1,0), (1,1), and (0,1), oriented counterclockwise.

a) 0

b) Positive

c) Negative



POLL 5

(Spring 22 Exam 2 #11)

Given the force field $\vec{\boldsymbol{F}}(x, y, z) = \langle y, z, x \rangle$, find the work required to move an object along the straight-line segment from (0,0,0) to (2,3,4).

a) 13

b) 9

c) 29

d) 26

e) 18

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Evaluating vector line integrals
b) Work
c) Flux
d) Circulation
e) None – understood everything today