LESSON 28

MA 26100·FALL 2023

DR. HOOD

WARM UP

(Spring 22 Exam 2 #11)

Given the force field $\vec{F}(x,y,z) = \langle y, z, x\rangle$, find the work required to move an object along the straight-line segment from (0,0,0) to (2,3,4).

$\vec{v} = \langle 2,3,4\rangle$

$\vec{r}(t) = \langle 2t, 3t, 4t\rangle \quad 0 \le t \le 1$

a) 13
b) 9
c) 29
d) 26
e) 18

$W = \int_C \vec{F}\cdot d\vec{r} = \int_0^1 \langle 3t, 4t, 2t\rangle \cdot \langle 2,3,4\rangle \, dt$

$= \int_0^1 6t + 12t + 8t \, dt = 26\int_0^1 t\,dt$

$= 26\left[\frac{t^2}{2}\right]_0^1 = 13$

LESSON 28

ANNOUNCEMENTS

- **Exam 2 is Tue Nov 7 at 8:00 – 9:00pm**
  - Similar format as Exam 1
  - Study Guide and Practice Exams posted on Brightspace

- **HW 29 is now due Fri Nov 10 at 11:59pm**
  - Course calendar is now up to date

POLL 1

Using the Fundamental Theorem of Line Integrals, evaluate $\int_C \vec{\nabla} f \cdot d\vec{r}$ where $f(x,y) = \frac{x^2}{y}$ and C is the curve parameterized by $\vec{r}(t) = \langle t^2, t \rangle$ for $1 \le t \le e$

$a = 1 \quad b = e$

$\int_C \vec{\nabla} f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

$= f(\vec{r}(e)) - f(\vec{r}(1))$

$= f(\langle e^2, e \rangle) - f(\langle 1, 1 \rangle)$

$= \frac{(e^2)^2}{e} - \frac{1^2}{1} = e^3 - 1$

a) $e^4 - 1$

b) $e^3 - 1$

c) e^3

d) e^4

POLL 2

Which of the following vector fields $\vec{\boldsymbol{F}}(x,y)$ is **not** conservative?

$\vec{F} = \langle P, Q \rangle \qquad \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

a) $\vec{\boldsymbol{F}}(x,y) = \langle x, y \rangle$

b) $\vec{\boldsymbol{F}}(x,y) = \langle -y, x \rangle$

$\frac{\partial}{\partial y}(-y) = -1 \overset{?}{=} \frac{\partial}{\partial x}(x) = 1$

$\times$ NOT

c) $\vec{\boldsymbol{F}}(x,y) = \langle 2xy, x^2 \rangle$

$\langle P, Q \rangle$

$2x = \frac{\partial}{\partial y}(2xy) = \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}(x^2) = 2x$ ✓

is conservative

POLL 3

Which of the following is a potential function f for the vector field $\vec{F} = \langle e^x \cos(y), -e^x \sin(y) + 3y^2 \rangle$?

a) $f = -e^x \sin(y) + y^3$

b) $f = e^x \cos(y) + 1$

c) $f = e^x \cos(y) + y^3 + 5$

$\int e^x \cos(y)\,dx = e^x \cos(y) + a(y)$

$\int (-e^x \sin(y) + 3y^2)\,dy$

$= e^x \cos(y) + y^3 + \underset{\substack{\| \\ c}}{b(x)}$

$f(x,y) = e^x \cos(y) + y^3 + c$

POLL 4

(Spring 23 Final Exam #11)

Let $\vec{\boldsymbol{F}}(x, y, z) = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z \rangle$, you may assume $\vec{\boldsymbol{F}}$ is conservative. Calculate the work done by $\vec{\boldsymbol{F}}$ moving an object along C where C is the straight-line segment from (0,0,2) to (0,3,0).

$a)\ 0$

$b)\ -9$

$c)\ 5$

$d)\ 12$

$e)\ -4$

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Fundamental Theorem of Line Integrals
b) Independent of Path
c) Checking if a vector field is conservative
d) Finding the potential function
e) None – understood everything today