LESSON 29

MA 26100·FALL 2023

DR. HOOD

WARM UP

(Spring 23 Final Exam #11)

Let $\vec{F}(x,y,z) = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z \rangle$, you may assume $\vec{F}$ is conservative. Calculate the work done by $\vec{F}$ moving an object along C where C is the straight-line segment from (0,0,2) to (0,3,0).

a) 0

b) −9

c) 5

d) 12

e) −4

$\vec{F} = \vec{\nabla}\phi \quad W = \int_C \vec{F}\cdot d\vec{r} = \phi(b) - \phi(a)$

$\phi = \int (3x^2yz - 3y)\,dx = x^3yz - 3xy + a(y,z)$

$= \int (x^3z - 3x)\,dy = x^3yz - 3xy + b(x,z)$

$= \int (x^3y + 2z)\,dz = x^3yz + z^2 + c(x,y)$

$\phi = x^3yz - 3xy + z^2 + C$

$W = \int_C \vec{F}\cdot d\vec{r} = \phi(0,3,0) - \phi(0,0,2) = 0 - 2^2 = -4$

LESSON 29

POLL 1

Let $\vec{\boldsymbol{F}}(x, y) = \vec{\nabla}\varphi$ be a conservative vector field. What is the curl $\vec{\nabla} \times \vec{\boldsymbol{F}}$?

a) 1

b) 0

c) $\frac{\partial^2\varphi}{\partial x^2} - \frac{\partial^2\varphi}{\partial y^2}$

d) $\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}$

$$\vec{F} = \vec{\nabla}\phi = \left\langle \frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y} \right\rangle$$

$$\vec{\nabla} \times \vec{F} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x}\right)$$

$$= \frac{\partial^2\phi}{\partial x \partial y} - \frac{\partial^2\phi}{\partial y \partial x} = 0$$

POLL 2

Use Green's Theorem to calculate the work done on an object by the force field $\vec{\boldsymbol{F}}(x,y) = \langle y + \sin(x), e^y - x \rangle$ on the path C that is the circle $x^2 + y^2 = 4$ starting at the point $(2,0)$ and making exactly one full loop.

a) 0

b) -4π

c) -8π

$$W = \oint_C \vec{F} \cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$= \iint_D \frac{\partial}{\partial x}\left(e^y - x\right) - \frac{\partial}{\partial y}\left(y + \sin(x)\right) dA$$

$$= \iint_D -1 - 1 \, dA = -2 \iint_D dA = -2\,\text{area}(D)$$

$$= -2(\pi 2^2) = \boxed{-8\pi}$$

POLL 3

Which of the following vector fields has $\vec{\nabla} \times \vec{\boldsymbol{F}} = 1$?

$a)\ \vec{\boldsymbol{F}}(x, y) = \langle 0, -x \rangle$

$b)\ \vec{\boldsymbol{F}}(x, y) = \langle x, y \rangle$

$c)\ \vec{\boldsymbol{F}}(x, y) = \left\langle -\frac{y}{2}, \frac{x}{2} \right\rangle$

POLL 4

Use the flux form of Green's Theorem to calculate the flux

$\oint_C \vec{\boldsymbol{F}} \cdot \overrightarrow{\boldsymbol{N}}\, ds$ of $\vec{\boldsymbol{F}}(x, y) = \langle x, y\rangle$, across the circle of radius 5.

$a)$ 0

$b)$ 5π

$c)$ 50π

POLL 5

(Fall 22 Final Exam #5)

5. Consider the circle C centered at 0 with radius 3. A particle travels once around C, counterclockwise. It is subject to the force

$$\mathbf{F}(x,y) = \langle y^3,\, x^3 + 3xy^2 + 1 \rangle \,.$$

Use Green's theorem to find the work done by $\mathbf{F}$.

A. $\frac{3\pi}{4}$

B. $\frac{4\pi}{3}$

C. $\frac{243\pi}{4}$

D. $\frac{117\pi}{4}$

E. $\frac{23\pi}{3}$