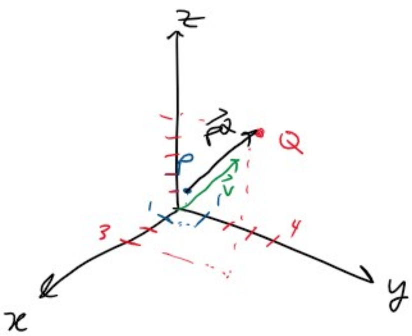
✩ Vectors in 3D space

xyz - coordinate system



$P(1,1,2)$
$Q(3,4,5)$

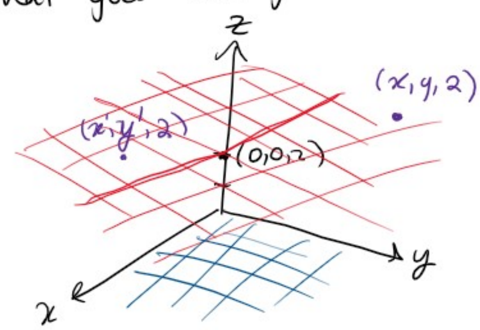
$\vec{PQ}$

position vector equal to $\vec{PQ}$

$$\vec{v} = \langle 3-1,\ 4-1,\ 5-2 \rangle = \langle 2, 3, 3 \rangle$$

$$|\vec{v}| = \sqrt{2^2 + 3^2 + 3^2} = \sqrt{4+9+9} = \sqrt{22}$$

Q: What is the equation of the plane parallel to the xy-plane that goes through $(0,0,2)$



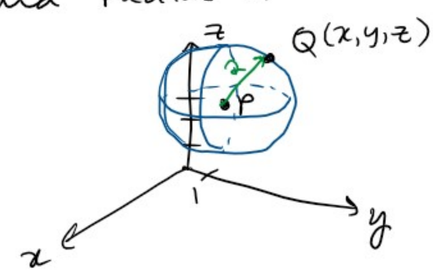
represent plane by

$$\boxed{z = 2}$$

The xy plane is given by $\boxed{z=0}$

The yz-plane is given by $\boxed{x=0}$

Q: What is the equation of the sphere with center at $(0,1,3)$ and radius 2



$P(0,1,3)$

point on sphere $Q(x,y,z)$

$$|\vec{PQ}| = 2$$

$$|\langle x-0,\ y-1,\ z-3 \rangle| = 2$$

$$\left(\sqrt{(x-0)^2 + (y-1)^2 + (z-3)^2}\right)^2 = (2)^2$$

$$\boxed{(x-0)^2 + (y-1)^2 + (z-3)^2 = 2^2}$$

↑ radius is 2

$$\boxed{(x-0)^2 + (y-1)^2 + (z-3)^2 = 2^2}$$

$\underbrace{\qquad\qquad}_{\text{center is } (0,1,3)}$ ↑ radius is 2

$$x^2 + y^2 - 2y + 1 + z^2 - 6z + 9 = 4$$

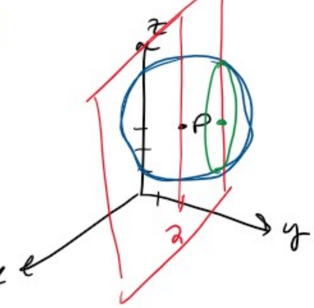
both of these represent a sphere

$$\boxed{x^2 + y^2 + z^2 - 2y - 6z = 4 - 1 - 9 = -6}$$

Q: Describe the points (x, y, z) that lie on the intersection of the sphere

$$(x-0)^2 + (y-1)^2 + (z-3)^2 = 2^2 \qquad P(0,1,3)$$

and the plane $y = 2$



plug $y=2$ into the eqn

$$(x-0)^2 + (y-1)^2 + (z-3)^2 = 2^2$$

$$(x-0)^2 + (2-1)^2 + (z-3)^2 = 2^2$$

$$(x-0)^2 + 1^2 + (z-3)^2 = 4$$

$$\boxed{\begin{gathered}(x-0)^2 + (z-3)^2 = 3 = r^2 \\ y = 2\end{gathered}}$$

← circle center at $(0, 2, 3)$ with radius $\sqrt{3}$

---

unit vectors in 3D:

$\vec{v} = \langle 1, -3, 4 \rangle$

unit vector $\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{\langle 1, -3, 4 \rangle}{\sqrt{1^2 + (-3)^2 + 4^2}}$

$$= \frac{\langle 1, -3, 4 \rangle}{\sqrt{1+9+16}} = \frac{\langle 1, -3, 4 \rangle}{\sqrt{26}}$$

$$= \boxed{\left\langle \frac{1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{4}{\sqrt{26}} \right\rangle}$$

Special unit vectors:

$\vec{i} = \hat{i} = \langle 1, 0, 0 \rangle$ — unit vector in x-direction

$\vec{j} = \hat{j} = \langle 0, 1, 0 \rangle$ — " " y-direction

$\vec{k} = \hat{k} = \langle 0, 0, 1 \rangle$ — " " z-direction

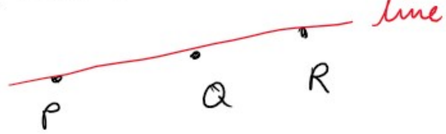
---

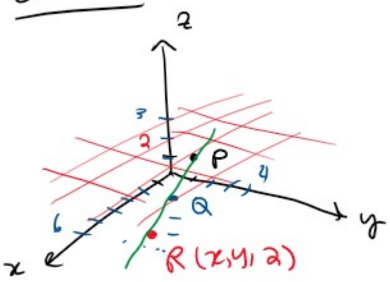
... and u so that

$k = k - \langle 0,$

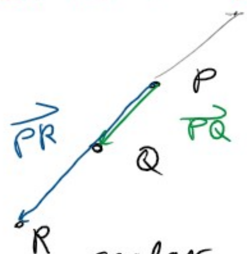
Q: Find all x and y so that $P(1,1,1)$, $Q(6,4,3)$ and $R(x,y,2)$ are collinear

collinear – P, Q, R all lie on the same line





R is on the z=2 plane



collinear if

$\vec{PQ} \parallel \vec{PR}$

$\vec{PQ} = k\,\vec{PR}$ where k is a scalar constant

$\vec{PR}$

$(x, y, 2) - (1, 1, 1)$

$= \langle$

$$\underbrace{\langle 6-1,\ 4-1,\ 3-1 \rangle}_{\vec{PQ}} = k \underbrace{\langle x-1,\ y-1,\ 2-1 \rangle}_{\vec{PR}}$$

$$\langle 5, 3, 2 \rangle = k \langle x-1,\ y-1,\ 1 \rangle = \langle k(x-1),\ k(y-1),\ k \rangle$$

separate out as 3 equations

$5 = k(x-1)$

$3 = k(y-1)$

$2 = k$

3 equations

3 unknowns x, y, k

$k = 2$

$5 = 2(x-1)$

$5 = 2x - 2$

$7 = 2x$

$\boxed{x = \frac{7}{2}}$

$3 = 2(y-1)$

$3 = 2y - 2$

$5 = 2y$

$\boxed{y = \frac{5}{2}}$

$\boxed{\text{P, Q, R are collinear when } R\left(\frac{7}{2}, \frac{5}{2}, 2\right)}$