LESSON 30

MA 26100·FALL 2023

DR. HOOD

WARM UP

LESSON 30

(Fall 22 Final Exam #5)

$$W = \oint_C \vec{F} \cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

5. Consider the circle C centered at 0 with radius 3. A particle travels once around C, counterclockwise. It is subject to the force

$$\mathbf{F}(x,y) = \langle \underbrace{y^3}_{P}, \underbrace{x^3 + 3xy^2 + 1}_{Q} \rangle .$$

Use Green's theorem to find the work done by $\mathbf{F}$.

A. $\frac{3\pi}{4}$

B. $\frac{4\pi}{3}$

C. $\frac{243\pi}{4}$

D. $\frac{117\pi}{4}$

E. $\frac{23\pi}{3}$

$$= \iint_D (3x^2 + \cancel{3y^2} - \cancel{3y^2})\, dA = \int_0^{2\pi} \int_0^3 3r^2 \cos^2\theta \, r\, dr\, d\theta$$

$$= 3 \int_0^{2\pi} \cos^2\theta \left[\frac{r^4}{4} \right]_0^3 d\theta = \frac{3^5}{4} \int_0^{2\pi} \frac{1 + \cos(2\theta)}{2} d\theta$$

$$= \frac{3^5}{2 \cdot 4} \left[\theta + \frac{\sin(2\theta)}{2} \right]_0^{2\pi} = \frac{3^5}{\cancel{2} \cdot 4} \cdot \cancel{2}\pi$$

ANNOUNCEMENTS

- **There will be class on Friday Nov 10**
  - Instead, class on Mon Nov 20 will be cancelled
  - Recitation on Tue Nov 21 is also cancelled
- **HW 29 and HW30 due Fri Nov 10 at 11:59pm**
- **Pearson is back online**
  - You can always check the status here: https://status.pearson.com/

POLL 1

Here is a vector field $\vec{F}(x, y)$. If you put a square in the first quadrant and let it change size according to the vector field, would the square expand or contract?

a) Expand

b) Contract

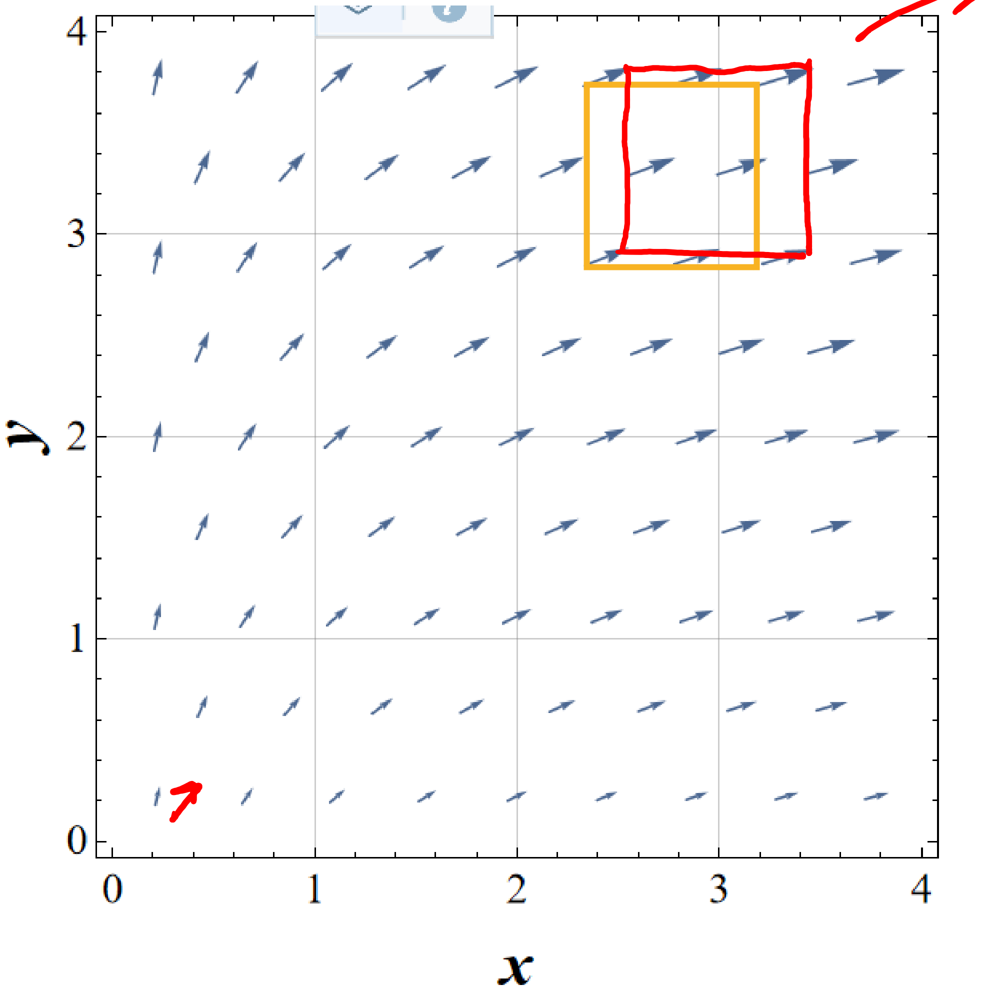
c) Neither

$\vec{F} = \langle xy, y \rangle$

$\vec{\nabla} \cdot F = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(y)$

$= y + 1 \quad > 0$

if $y \geq 0$

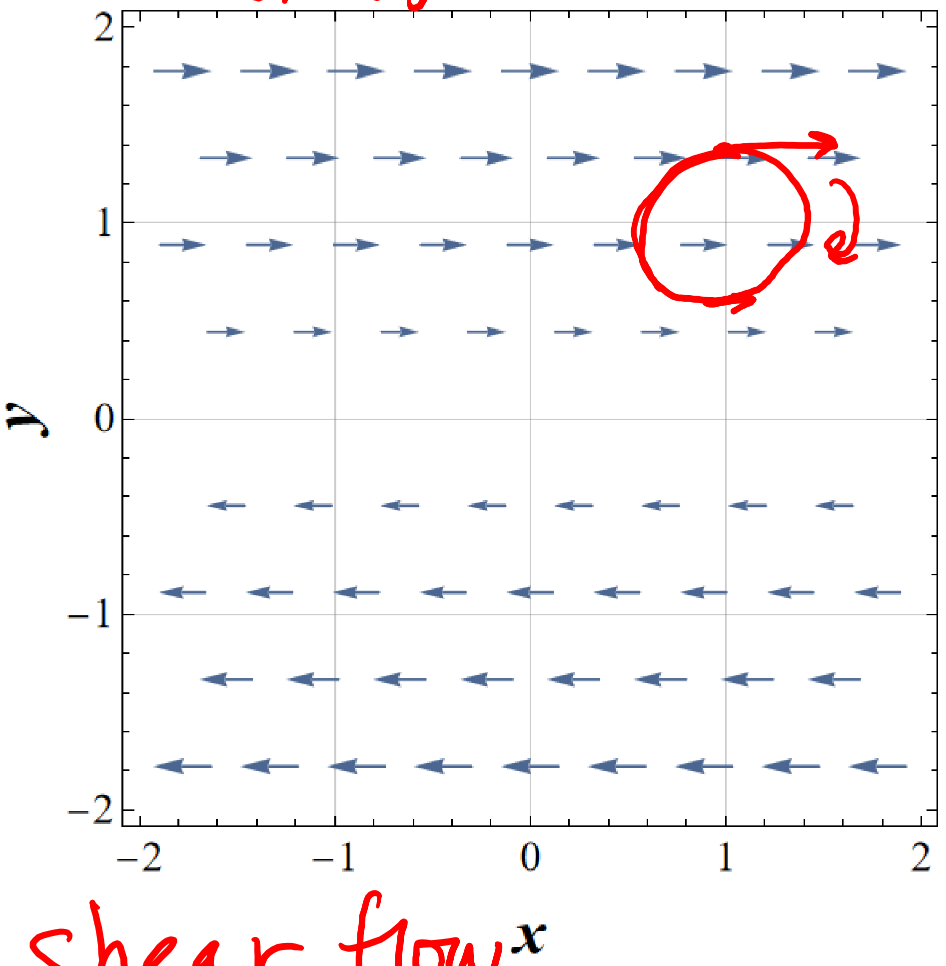


POLL 2

Which of the following vector fields is irrotational?

a) $\vec{F}(x,y) = \langle y, 0 \rangle$

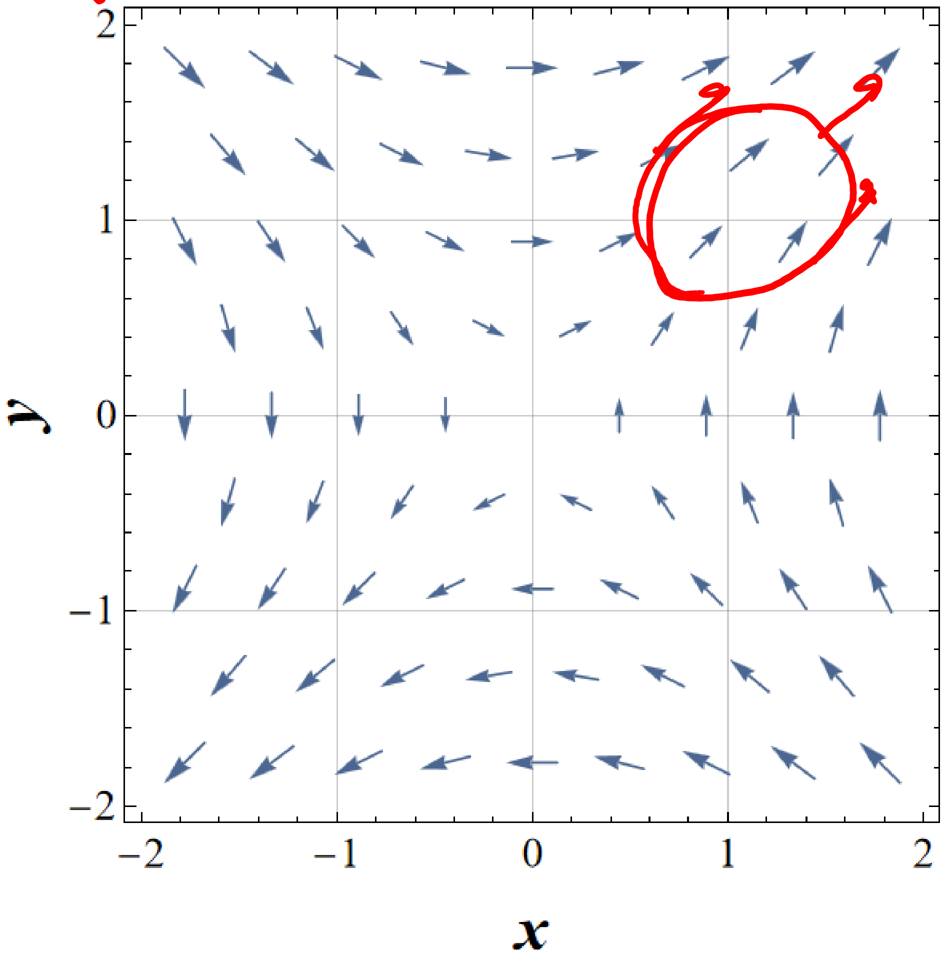
$\vec{\nabla} \times \vec{F} = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0 - (1) = -1$



shear flow

b) $\vec{F}(x,y) = \langle 3y, 3x \rangle$

$\vec{\nabla} \times \vec{F} = 3 - 3 = 0$

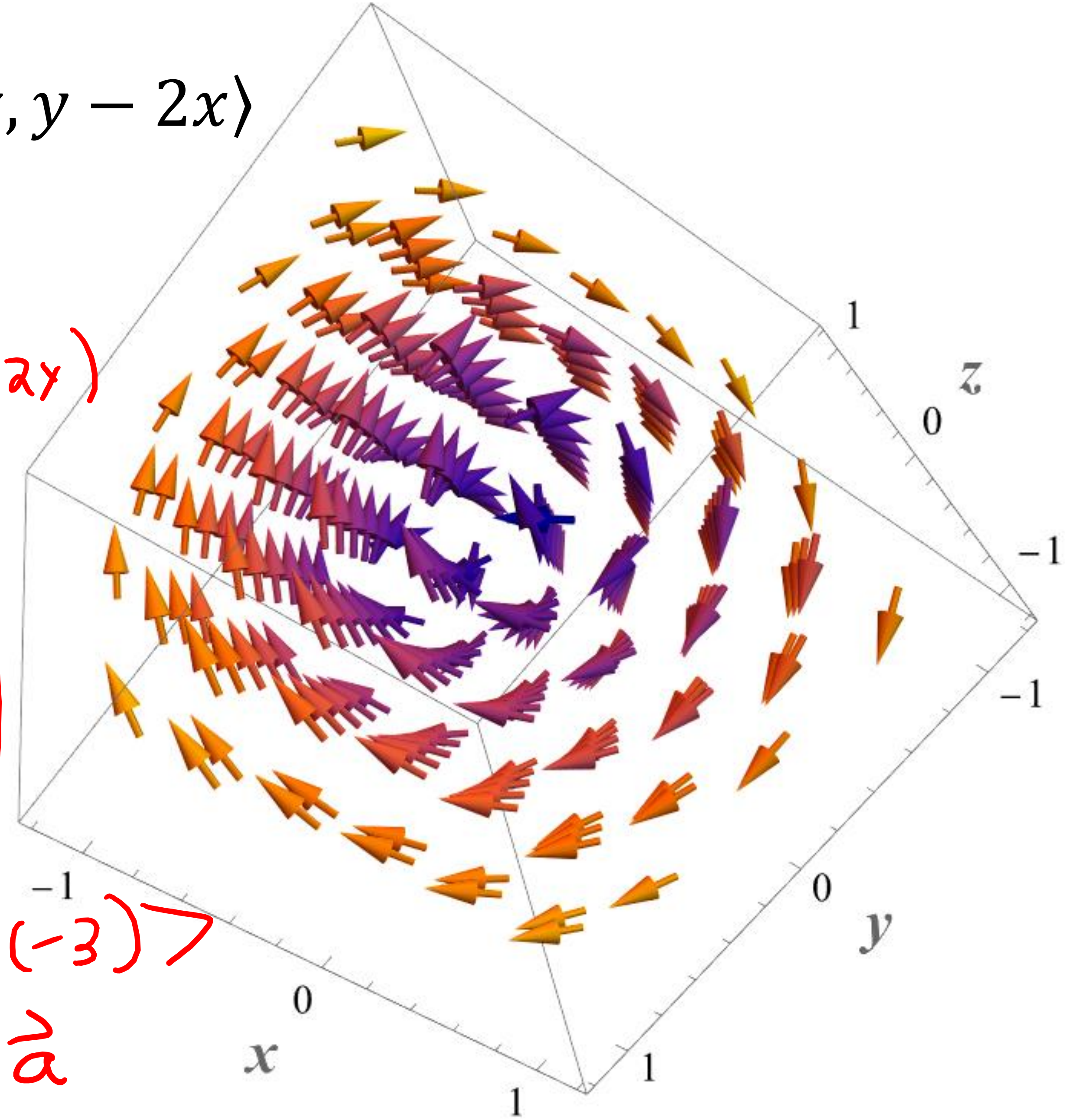


EXAMPLE

General rotational field

$\vec{a} = \langle 1, 2, 3 \rangle$

$$\vec{F}(x,y,z) = \langle 2z - 3y, 3x - z, y - 2x \rangle$$



What is $\vec{\nabla} \cdot \vec{F}$?

$\frac{\partial}{\partial x}(2z-3y) + \frac{\partial}{\partial y}(3x-z) + \frac{\partial}{\partial z}(y-2x)$

$\vec{\nabla} \cdot \vec{F} = 0$

What is $\vec{\nabla} \times \vec{F}$?

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2z-3y & 3x-z & y-2x \end{vmatrix}$$

$$= \langle 1-(-1), 2-(-2), 3-(-3) \rangle$$

$$= 2\langle 1, 2, 3 \rangle = 2\vec{a}$$

POLL 3

Let $\vec{\boldsymbol{F}}(x,y,z) = \langle P, Q, R\rangle$. Find $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{\boldsymbol{F}}\right)$

a) 1

b) 0

c) $\dfrac{\partial^2 P}{\partial x^2} + \dfrac{\partial^2 Q}{\partial y^2} + \dfrac{\partial^2 R}{\partial z^2}$

d) $2\dfrac{\partial^2 P}{\partial z \partial y} + 2\dfrac{\partial^2 Q}{\partial x \partial z} + 2\dfrac{\partial^2 R}{\partial y \partial x}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = \frac{\partial}{\partial x}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

$$= \cancel{\frac{\partial^2 R}{\partial x \partial y}} - \cancel{\frac{\partial^2 Q}{\partial x \partial z}} + \cancel{\frac{\partial^2 P}{\partial y \partial z}} - \cancel{\frac{\partial^2 R}{\partial y \partial x}} + \cancel{\frac{\partial^2 Q}{\partial z \partial x}} - \cancel{\frac{\partial^2 P}{\partial z \partial y}} = 0$$

POLL 4

(Fall 22 Final Exam #19)

19. Let $\mathbf{F} = \langle ax, cz - ax, cz + by \rangle$ be a vector field, where $a, b, c \in \mathbb{R}$. Find conditions on a, b and c so that $\mathbf{F}$ is **not** conservative and such that curl($\mathbf{F}$) is parallel to $\mathbf{k}$.

A. $b = c$

B. $a = c$

C. $b = c$ and $a = 0$

D. $b = c$ and $a \neq 0$

E. $a = b = c = 0$

POLL 5

(Fall 17 Final Exam #14)

14. Let $\mathbf{F} = \langle x^2yz,\, xy^2z,\, xyz^2\rangle$. Compute grad(div $\mathbf{F}$) – curl(curl($\mathbf{F}$)).

A. $\langle 2yz,\, 2xz,\, 2xy\rangle$

B. $\langle 0,\, 0,\, 0\rangle$

C. $\langle 6yz,\, 6xz,\, 6xy\rangle$

D. $\langle 4yz,\, 4xz,\, 4xy\rangle$

E. $\langle yz,\, xz,\, xy\rangle$