LESSON 31

MA 26100·FALL 2023

DR. HOOD

WARM UP

LESSON 31

(Fall 22 Final Exam #19)

$\vec{\nabla} \times \vec{F} \parallel \hat{k}$

19. Let $\mathbf{F} = \langle ax, cz - ax, cz + by \rangle$ be a vector field, where $a, b, c \in \mathbb{R}$. Find conditions on a, b and c so that $\mathbf{F}$ is **not** conservative and such that curl($\mathbf{F}$) is parallel to $\mathbf{k}$.

A. $b = c$

B. $a = c$

C. $b = c$ and $a = 0$

D. $b = c$ and $a \neq 0$

E. $a = b = c = 0$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ ax & cz - ax & cz + by \end{vmatrix}$$

$$= \langle b - c,\ 0 - 0,\ -a - 0 \rangle$$

$$\langle b - c, 0, -a \rangle = \lambda \langle 0, 0, 1 \rangle$$

$b - c = \lambda 0$ $\quad\quad$ $-a = \lambda$

$b = c$ $\quad\quad$ $a \neq 0$

ANNOUNCEMENTS

- **Exam 2 Scores returned next week**
  - Hopefully by the end of the day Tuesday 🤞

POLL 1

Parameterize the surface of the cylinder $x^2 + y^2 = 9$ from $z = 0$ to $z = 1$. Hint: Cylindrical Coordinates

a) $\vec{r}(u,v) = \langle 3\cos(u), 3\sin(u), v\rangle$ for $0 \le u \le 2\pi$, $0 \le v \le 1$

b) $\vec{r}(u,v) = \langle 3\sin(u), 3\cos(u), v\rangle$ for $0 \le u \le 1$, $0 \le v \le 2\pi$

c) $\vec{r}(u,v) = \langle 3\sin(v)\cos(u), 3\sin(v)\sin(u), 3\cos(v)\rangle$ for $0 \le u \le 2\pi$, $0 \le v \le \pi$

$x^2 + y^2 = 9 \longrightarrow r = 3$

$0 \le z \le 1$ ($z = v$)

$0 \le \theta \le 2\pi$ ($\theta = u$)

$\vec{r}(u,v) = \langle 3\cos(u), 3\sin(u), v \rangle$

POLL 2

Given $\vec{r}(u,v) = \langle 3\cos(u), 3\sin(u), v\rangle$ for $0 \le u \le 2\pi$, $0 \le v \le 1$, find $\left|\vec{t}_u \times \vec{t}_v\right|$.

a) 3

b) 9

c) 6

$\vec{t}_u = \langle -3\sin(u), 3\cos(u), 0 \rangle$

$\vec{t}_v = \langle 0, 0, 1 \rangle$

$$|\vec{t}_u \times \vec{t}_v| = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ -3\sin(u) & 3\cos(u) & 0 \\ 0 & 0 & 1 \end{matrix} \right|$$

$$= |\langle 3\cos(u), 3\sin(u), 0 \rangle|$$

$$= \sqrt{3^2\cos^2(u) + 3^2\sin^2(u)} = 3$$

POLL 3

(Spring 23 Final Exam #15)

The surface parameterized by

$$\vec{\boldsymbol{r}}(u, v) = \langle 5\sin(v)\cos(u)\,, 5\sin(v)\sin(u)\,, 5\cos(v)\rangle$$

is a:

a) Plane
b) Ellipse
c) Sphere
d) Cylinder
e) Paraboloid

POLL 4

(Fall 13 Final Exam #19)

A normal vector to the surface

$$\vec{\boldsymbol{r}}(u,v) = \langle \sin(u), \sin(v)\cos(v), \sin(v) \rangle$$

At $(u,v) = \left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ is:

a) $\langle -3, -2, 1 \rangle$

b) $\left\langle -\frac{1}{8}, \frac{1}{4}, 1 \right\rangle$

c) $\langle 3, 2, 1 \rangle$

d) $\left\langle -\frac{3}{8}, \frac{1}{4}, 2 \right\rangle$

e) $\left\langle \frac{3}{8}, \frac{1}{4}, 2 \right\rangle$