# LESSON 32

## MA 26100·FALL 2023

DR. HOOD

WARM UP

(Spring 23 Final Exam #15)

The surface parameterized by

$$\vec{r}(u, v) = \langle 5\sin(v)\cos(u), 5\sin(v)\sin(u), 5\cos(v)\rangle$$

is a:

$x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta \quad z = \rho\cos\phi$

a) Plane

b) Ellipse

c) Sphere

d) Cylinder

e) Paraboloid

$\phi = v$

$\theta = u$

$\rho = 5$

LESSON 32

# ANNOUNCEMENTS

- Exam 2 Scores will be returned by the end of the day Tuesday

POLL 1

For the cone parameterized by $\vec{r}(u,v) = \langle v\cos(u), v\sin(u), v\rangle$ for $0 \le u \le 2\pi$, $0 \le v \le 3$, find $\left|\vec{t}_u \times \vec{t}_v\right|$.

a) $v$

b) $\sqrt{1+v^2}$

c) $\sqrt{2}\,v$

$\vec{t}_u = \langle -v\sin(u), v\cos(u), 0\rangle$

$\vec{t}_v = \langle \cos(u), \sin(u), 1\rangle$

$$|\vec{t}_u \times \vec{t}_v| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -v\sin(u) & v\cos(u) & 0 \\ \cos(u) & \sin(u) & 1 \end{vmatrix} = \langle v\cos(u), v\sin(u), -v\sin^2(u) - v\cos^2(u)\rangle$$

$$= |\langle v\cos(u), v\sin(u), v\rangle|$$

$$= \sqrt{\underbrace{v^2\cos^2(u) + v^2\sin^2(u)} + v^2}$$

$$= \sqrt{v^2 + v^2} = v\sqrt{2}$$

POLL 2

(Fall 22 Final Exam #20)

$\vec{r}(u,v) = \langle u, v, 8-2u-v \rangle \quad 0 \le u, v \le 1$

Find the mass of the part of the plane $z = 8 - 2x - y$ that lies over the square $[0,1] \times [0,1]$ when the density function is given by $f(x, y, z) = 12 - z$.

$\vec{r}_u = \langle 1, 0, -2 \rangle \qquad \vec{r}_v = \langle 0, 1, -1 \rangle$

a) 12

b) $11\sqrt{6}$

c) $\sqrt{6}$

d) $\frac{11}{2}$

e) $\frac{11\sqrt{6}}{2}$

$|\vec{r}_u \times \vec{r}_v| = \left| \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & -2 \\ 0 & 1 & -1 \end{matrix} \right| = |\langle 2, 1, 1 \rangle| = \sqrt{6}$

$\text{mass} = \int_0^1 \int_0^1 (12 - (8 - 2u - v))\sqrt{6}\, du\, dv$

$= \sqrt{6} \int_0^1 \int_0^1 (4 + 2u + v)\, du\, dv = \int_0^1 \left[ 4u + u^2 + uv \right]_0^1 dv$

$= \sqrt{6} \int_0^1 (5 + v)\, dv = \sqrt{6} \left[ 5v + \frac{v^2}{2} \right]_0^1$

$= \sqrt{6} \left[ 5 + \frac{1}{2} \right] = \frac{11\sqrt{6}}{2}$

POLL 3

Parameterize the elliptic paraboloid $z = 16 - x^2 - y^2$ using Cartesian coordinates and find $|\vec{t}_u \times \vec{t}_v|$.

a) $\sqrt{1 + 4u^2 + 4v^2}$

b) $\sqrt{1 - 4u^2 - 4v^2}$

c) $2\sqrt{u^2 + v^2}$

d) $\sqrt{1 + u^2 + v^2}$

$\vec{r}(u,v) = \langle u, v, 16 - u^2 - v^2 \rangle$

$\vec{t}_u = \langle 1, 0, -2u \rangle$

$\vec{t}_v = \langle 0, 1, -2v \rangle$

$|\vec{t}_u \times \vec{t}_v| = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -2u \\ 0 & 1 & -2v \end{matrix} \right| = |\langle 2u, 2v, 1 \rangle|$

$= \sqrt{1^2 + 4u^2 + 4v^2}$

$= \sqrt{1 + 4u^2 + 4v^2}$

POLL 4

# (Fall 15 Final Exam #16)

PROBLEM 16: Let $S$ be the piece of the surface $z = xy + 5$ that lies inside the cylinder $x^2 + y^2 = 4$ with upward normal. Calculate

$$\iint_S 2\, dS$$

A. $\frac{4\pi}{3}(5\sqrt{5} - 1)$

B. $\frac{2\pi}{3}(5\sqrt{5} - 1)$

C. $\frac{8\pi}{3}(5\sqrt{5} - 1)$

D. $\frac{4\pi}{3}$

E. $\frac{2\pi}{3}(\sqrt{5} - 1)$