LESSON 33

MA 26100·FALL 2023

DR. HOOD

WARM UP

LESSON 33

(Spring 1999 Final Exam #24)

$\vec{r}(u,v) = \langle u, v, u^2+v^2 \rangle$

Let S be the portion of the paraboloid $z = x^2 + y^2$ satisfying $z < 4$. Compute the surface integral:

$$\iint_S \left(1 + 4x^2 + 4y^2\right)^{-\frac{1}{2}} dS$$

a) 16π

b) 2π

c) $\pi(\sqrt{17} - 1)$

d) 4π

e) $2\pi\sqrt{17}$

$t_u = \langle 1, 0, 2u \rangle$

$t_v = \langle 0, 1, 2v \rangle$

$|t_u \times t_v| = |\langle -2u, -2v, 1 \rangle|$

$= \sqrt{1 + 4u^2 + 4v^2}$

$\iint_D \frac{\sqrt{1+4u^2+4v^2}}{\sqrt{1+4u^2+4v^2}} dA = \text{area}(D)$

$= \pi(2)^2 = 4\pi$

ANNOUNCEMENTS

- Exam 2 Scores released
  - Average 79.4
  - Median 84
  - 34 students lost 2 points for incorrect PUID on scantron

POLL 1

(Fall 13 Final Exam #19)

A normal vector to the surface

$$\vec{r}(u,v) = \langle \sin(u), \cos(u)\sin(v), \sin(v) \rangle$$

At $(u,v) = \left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ is:

a) $\langle -3, -2, 1 \rangle$

b) $\left\langle -\frac{1}{8}, \frac{1}{4}, 1 \right\rangle$

c) $\langle 3, 2, 1 \rangle$

d) $\left\langle -\frac{3}{8}, \frac{1}{4}, 2 \right\rangle$

e) $\left\langle \frac{3}{8}, \frac{1}{4}, 2 \right\rangle$

$\vec{t}_u = \langle \cos(u), -\sin(u)\sin(v), 0 \rangle \Big|_{u=\frac{\pi}{3},\, v=\frac{\pi}{3}}$

$= \langle \frac{1}{2}, -\frac{3}{4}, 0 \rangle$

$\vec{t}_v = \langle 0, \cos(u)\cos(v), \cos(v) \rangle \Big|_{u=\frac{\pi}{3},\, v=\frac{\pi}{3}}$

$= \langle 0, \frac{1}{4}, \frac{1}{2} \rangle$

$$t_u \times t_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & -\frac{3}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} \end{vmatrix} = \langle -\frac{3}{8}, -\frac{1}{4}, \frac{1}{8} \rangle$$

$$8(t_u \times t_v) = \langle -3, -2, 1 \rangle$$

POLL 1

(Fall 13 Final Exam #19)

A normal vector to the surface

$$\vec{\boldsymbol{r}}(u,v) = \langle \sin(u), \cos(u)\sin(v), \sin(v) \rangle$$

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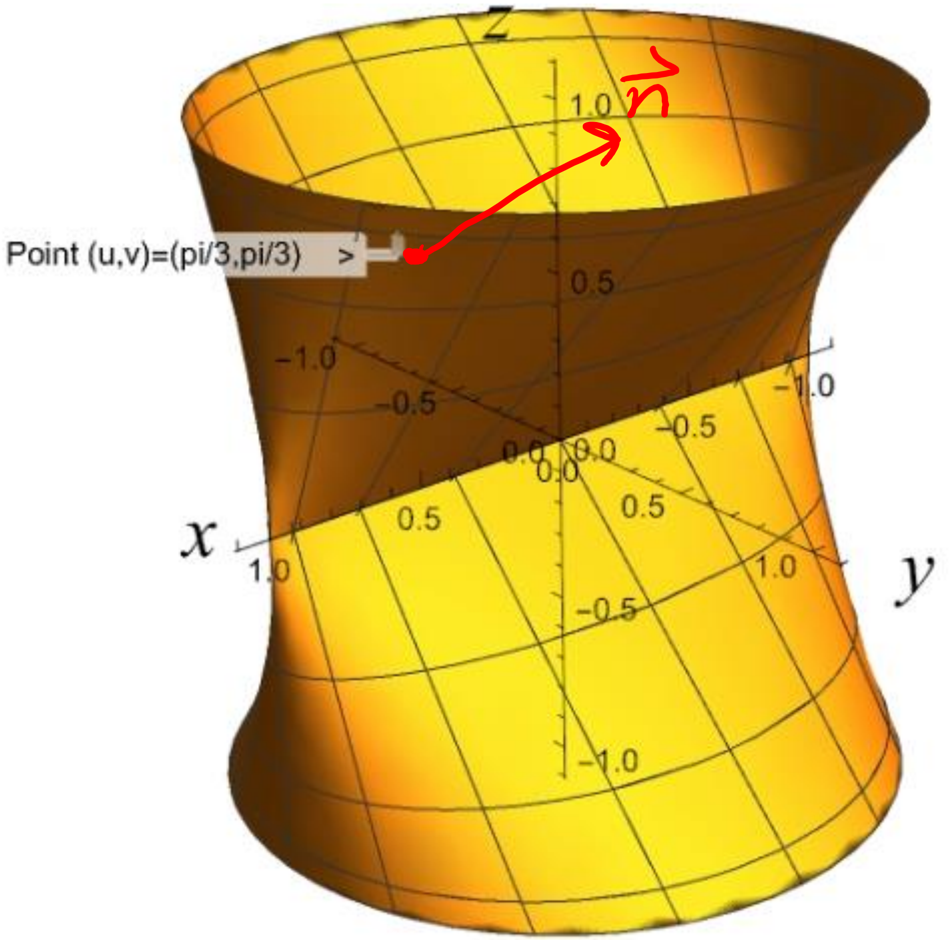

POLL 2

Set up the integral for $\iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, dS$ where $\vec{\boldsymbol{F}} = \langle x, y, z \rangle$ and S is the paraboloid $z = 1 - x^2 - y^2$ in the first octant.

a) $\int_0^1 \int_0^{\sqrt{1-u^2}} 1\, dv\, du$

b) $\int_0^1 \int_0^{\sqrt{1-u^2}} (u^2 + v^2 + 1)\, dv\, du$

c) $\int_0^1 \int_0^{\sqrt{1-u^2}} (2u + 2v + 1)\, dv\, du$

$\vec{r}(u,v) = \langle u, v, 1-u^2-v^2 \rangle$

$t_u = \langle 1, 0, -2u \rangle$

$t_v = \langle 0, 1, -2v \rangle$

$t_u \times t_v = \langle 2u, 2v, 1 \rangle$

$\iint_D \langle u, v, 1-u^2-v^2 \rangle \cdot \langle 2u, 2v, 1 \rangle\, dA$

$= \iint_D 2u^2 + 2v^2 + 1 - u^2 - v^2\, dA = \iint_D u^2 + v^2 + 1\, dA$

POLL 3

Set up the flux integral of the radial field $\vec{\boldsymbol{F}} = \langle x, y, z \rangle$ over the surface of the hemisphere $x^2 + y^2 + z^2 = 1$ for $z \geq 0$. (Hint: use implicit differentiation).

a) $\iint_D \frac{1}{\sqrt{1-x^2-y^2}}\, dA$

b) $\iint_D \frac{1}{1-x^2-y^2}\, dA$

c) $\iint_D \sqrt{1-x^2-y^2}\, dA$