LESSON 34

MA 26100·FALL 2023

DR. HOOD

WARM UP

(Spring 2016 Final Exam #18)

$\vec{n} = \langle 2, 1, -1 \rangle$

If $\vec{\boldsymbol{F}}(x, y, z) = \langle 1,2,1 \rangle$ and S is the intersection of the solid cylinder $x^2 + y^2 \leq 1$ with the plane $2x + y - z = 1$, compute $\iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, dS$ (using the upward pointing $\vec{\boldsymbol{n}}$).

$\vec{n} = \langle -2, -1, 1 \rangle$

a) $-\pi$

b) -3π

c) 2π

d) 3π

e) $\frac{\pi}{2}$

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iint_D \langle 1,2,1 \rangle \cdot \langle -2,-1,1 \rangle\, dA$$

$$= \iint_D (-2-2+1)\, dA = -3 \iint_D dA$$

$$= -3\,\text{area}(D) = -3\pi(1)^2$$

$$= -3\pi$$

LESSON 34

ANNOUNCEMENTS

- Class cancelled on Monday Nov 20
- Recitation cancelled on Tuesday Nov 21

POLL 1

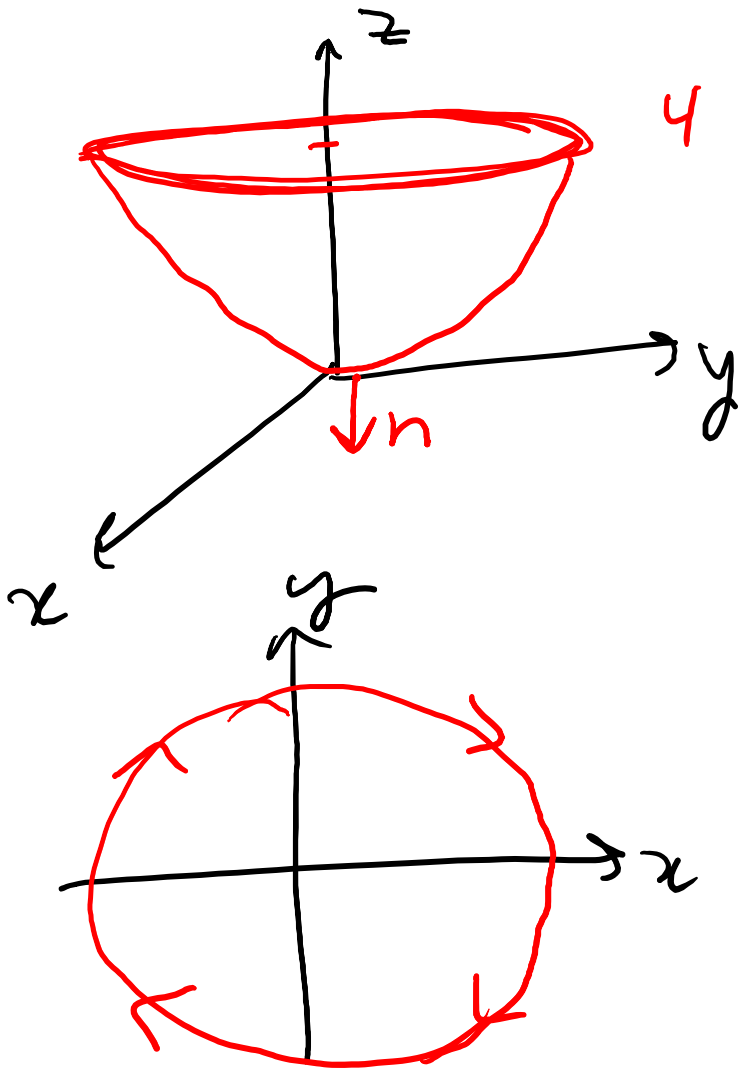
Let S be the surface of the paraboloid $z = x^2 + y^2$ for $z \leq 4$ oriented with $\vec{\boldsymbol{n}}$ pointing downward. Find C and parameterize it so that it has right-handed orientation.

a) $\vec{\mathbf{r}}(t) = \langle 2\cos(t), 2\sin(t), 4 \rangle$

b) $\vec{\mathbf{r}}(t) = \langle 2\cos(t), 4, -2\sin(t) \rangle$

c) $\vec{\mathbf{r}}(t) = \langle 2\cos(t), -2\sin(t), 4 \rangle$

d) $\vec{\mathbf{r}}(t) = \langle 2\cos(t), 4, 2\sin(t) \rangle$



POLL 2

Use Stokes' Theorem to rewrite the line integral as a surface integral: $\oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ where $\vec{\boldsymbol{F}} = \langle z^2, y^2, x \rangle$ and C is the triangle with vertices $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$ oriented with counterclockwise rotation.

a) $\int_0^1 \int_0^{1-u} (1 - 2u - 2v)\, dv\, du$

b) $\int_0^1 \int_0^1 \left((1 - u - v)^2 + v^2 + u\right) dv\, du$

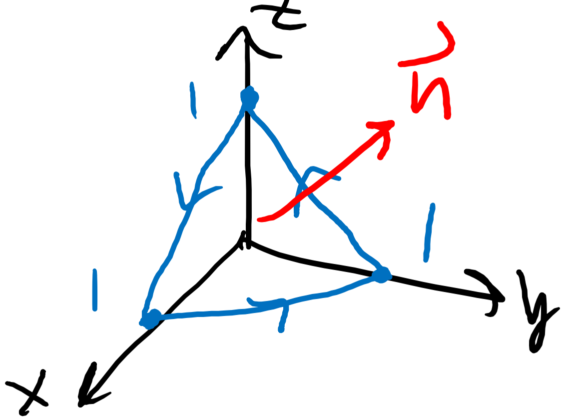
c) $\int_0^1 \int_0^{1-u} (2u + 2v)\, dv\, du$

$x + y + z = 1$

$\vec{n} = \langle 1, 1, 1 \rangle$

$\vec{r}(u,v) = \langle u, v, 1-u-v \rangle$

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z^2 & y^2 & x \end{vmatrix} = \langle 0, 2z-1, 0 \rangle = \langle 0, 2(1-u-v)-1, 0 \rangle$$



$$\iint_D \vec{\nabla} \times \vec{F} \cdot \vec{n}\, dA$$

$$= \iint_D (1-2u-2v)\, dA$$

POLL 3

(Fall 2022 Final Exam #10)

Given $\vec{\boldsymbol{F}} = \langle -y, x, z^3 \rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ above $z = 1$. Compute $\iint_S \left(\nabla \times \vec{\boldsymbol{F}}\right) \cdot d\vec{\boldsymbol{S}}$

a) -6π

b) 6π

c) 0

d) 3π

e) -3π