LESSON 35

MA 26100·FALL 2023

DR. HOOD

WARM UP

(Fall 2022 Final Exam #10)

Given $\vec{F} = \langle -y, x, z^3 \rangle$ and surface S is the part of the sphere $x^2 + y^2 + z^2 = 4$ above $z = 1$. Compute $\iint_S \left(\nabla \times \vec{F}\right) \cdot d\vec{S}$

Stokes' Theorem

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dS = \oint_C \vec{F} \cdot d\vec{r}$$

a) -6π

b) 6π

c) 0

d) 3π

e) -3π

where C is $x^2 + y^2 + 1^2 = 4$

$x^2 + y^2 = \sqrt{3}$

$\vec{r}(t) = \langle \sqrt{3}\cos(t), \sqrt{3}\sin(t), 1 \rangle$

$\vec{r}'(t) = \langle -\sqrt{3}\sin(t), \sqrt{3}\cos(t), 0 \rangle$

$$= \int_0^{2\pi} \langle -\sqrt{3}\sin(t), \sqrt{3}\cos(t), 1^3 \rangle \cdot \langle -\sqrt{3}\sin(t), \sqrt{3}\cos(t), 0 \rangle\, dt$$

$$= \int_0^{2\pi} 3\sin^2(t) + 3\cos^2(t) + 0\, dt = 3\int_0^{2\pi} dt$$

$$= 6\pi$$

LESSON 35

ANNOUNCEMENTS

- Exam 2 booklets returned in Recitation on Tue Nov 28

- HW 37 and Quiz 11 are optional and will not be graded

- ~~Grade Calculator: https://www.math.purdue.edu/~kthood/docs/MA261_Fall2023/grade_calculator_ma261_sp23.xlsx~~
  - There is an error – I will repost when it has been corrected

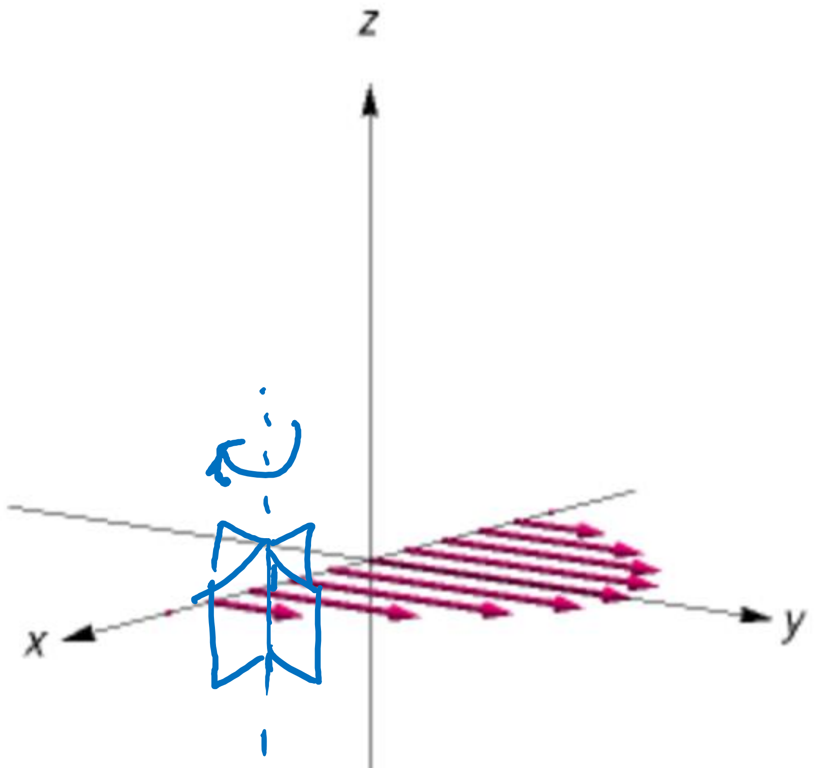
POLL 1

Consider $\vec{F} = \langle 0, 1 - x^2, 0 \rangle$ for $0 \leq x \leq 1$. You place a paddle wheel with its axis in the z-direction at the point $\left(\frac{1}{2}, 0, 0\right)$. Will it spin? If so, in which direction?

a) Does not spin

b) Spins clockwise

c) Spins counterclockwise



$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 1-x^2 & 0 \end{vmatrix}$$

$$= \langle 0, 0, -2x \rangle$$

@ $(\frac{1}{2}, 0, 0)$

$\nabla \times F = \langle 0, 0, -1 \rangle$

cw

POLL 2

Find $\iint_S \left(\nabla \times \vec{\boldsymbol{F}}\right) \cdot \vec{\boldsymbol{n}}\, dS$ where $\vec{\boldsymbol{F}} = \langle x, y, z \rangle$ and S is the surface of the plane $3x + 2y + z = 6$ in the first octant with upward pointing normal.

Hint: Calculate curl first

a) 0

b) 6

c) 9

d) 12

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$$

$$= \langle 0, 0, 0 \rangle$$

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dS = \iint_S 0 \cdot dS = 0$$

POLL 3

(Spring 2023 Final Exam #18)

Let S be an open surface in $\mathbb{R}^3$ with a boundary curve C where C is a circle in the xy-plane with radius 1 and center (0,0,0) and is oriented counterclockwise when viewed from above. Evaluate

$\oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ where $\vec{\boldsymbol{F}}(x, y, z)$ is such that $\nabla \times \vec{\boldsymbol{F}} = \left\langle \frac{1}{2}, -\frac{3}{2}, \frac{1}{4} \right\rangle$.

a) $\frac{\pi}{4}$

b) $\frac{\pi}{2}$

c) π

d) $-\frac{3\pi}{2}$

e) 2π

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dS$$

Disk : $z = 0$ $\vec{n} = \langle 0, 0, 1 \rangle$

$$\iint_D \left\langle \frac{1}{2}, -\frac{3}{2}, \frac{1}{4} \right\rangle \cdot \langle 0, 0, 1 \rangle \, dA$$

$$= \frac{1}{4} \iint_D dA = \frac{1}{4}\pi$$

POLL 4

(Spring 2017 Final Exam #19)

19. Let S be the semi-ellipsoid $z = 2\sqrt{1 - x^2 - y^2}$, oriented so that the normal $\mathbf{n}$ is upward pointing. Let

$$\mathbf{F}(x, y, z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2 \tan xy\,\mathbf{k}.$$

Use Stokes' Theorem to to evaluate $\int\int_S \mathrm{curl}\mathbf{F} \cdot \mathbf{n}\, dS$.

A. 2π

B. 4π

C. 0

D. $\frac{4\pi}{3}$

E. $\frac{2\pi}{3}$