LESSON 36

MA 26100·FALL 2023

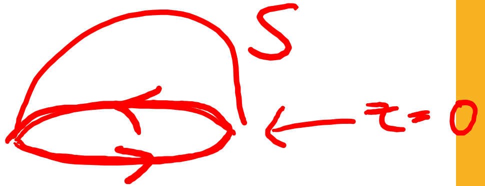
DR. HOOD

WARM UP

(Spring 2017 Final Exam #19)

$0 = 2\sqrt{1-x^2-y^2}$

$x^2+y^2=1$



19. Let S be the semi-ellipsoid $z = 2\sqrt{1-x^2-y^2}$, oriented so that the normal $\mathbf{n}$ is upward pointing. Let

$$\mathbf{F}(x,y,z) = x^2\mathbf{i} + y^2\mathbf{j} + z^2 \tan xy\,\mathbf{k}.$$

Use Stokes' Theorem to to evaluate $\int\int_S \text{curl}\mathbf{F}\cdot\mathbf{n}\,dS$.

A. 2π

B. 4π

C. 0

D. $\frac{4\pi}{3}$

E. $\frac{2\pi}{3}$

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\,dS = \oint_C \vec{F}\cdot d\vec{r}$$

parameterize $\vec{r}(t) = \langle \cos(t), \sin(t), 0\rangle$

$$= \int_0^{2\pi} \langle \cos^2(t), \sin^2(t), 0\rangle \cdot \langle -\sin(t), \cos(t), 0\rangle\,dt$$

$$= \int_0^{2\pi} -\sin(t)\cos^2(t) + \int_0^{2\pi} \sin^2(t)\cos(t)\,dt$$

$u = \cos(t)$ $\quad v = \sin(t)$

$$= \int_1^1 u^2\,du + \int_0^0 v^2\,dv = 0$$

LESSON 36

ANNOUNCEMENTS

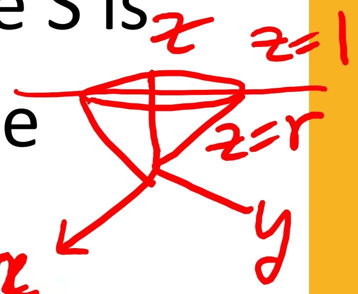
- Final Exam is Mon Dec 11 at 8:00am- 10:00am
  - Note: in the morning!

- Final Exam Study Guide: https://www.math.purdue.edu/~kthood/docs/MA261_Fall2023/final_study_guide_ma261_f23.pdf

POLL 1

$\nabla \cdot \vec{F} = 1 + 0 + 1 = 2$ $\qquad z = \rho\cos\phi = 1$

Consider $\vec{F} = \langle x - y, x + z, z - y\rangle$. Find $\iint_S \vec{F}\cdot\vec{n}\,dS$ where S is the surface of the cone $z^2 = x^2 + y^2$ for $0 \le z \le 1$ and the circular top of the cone.



a) $\frac{2\pi}{3}$

b) 4π

c) $\frac{4\pi}{3}$

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_E \nabla\cdot\vec{F}\,dV$$

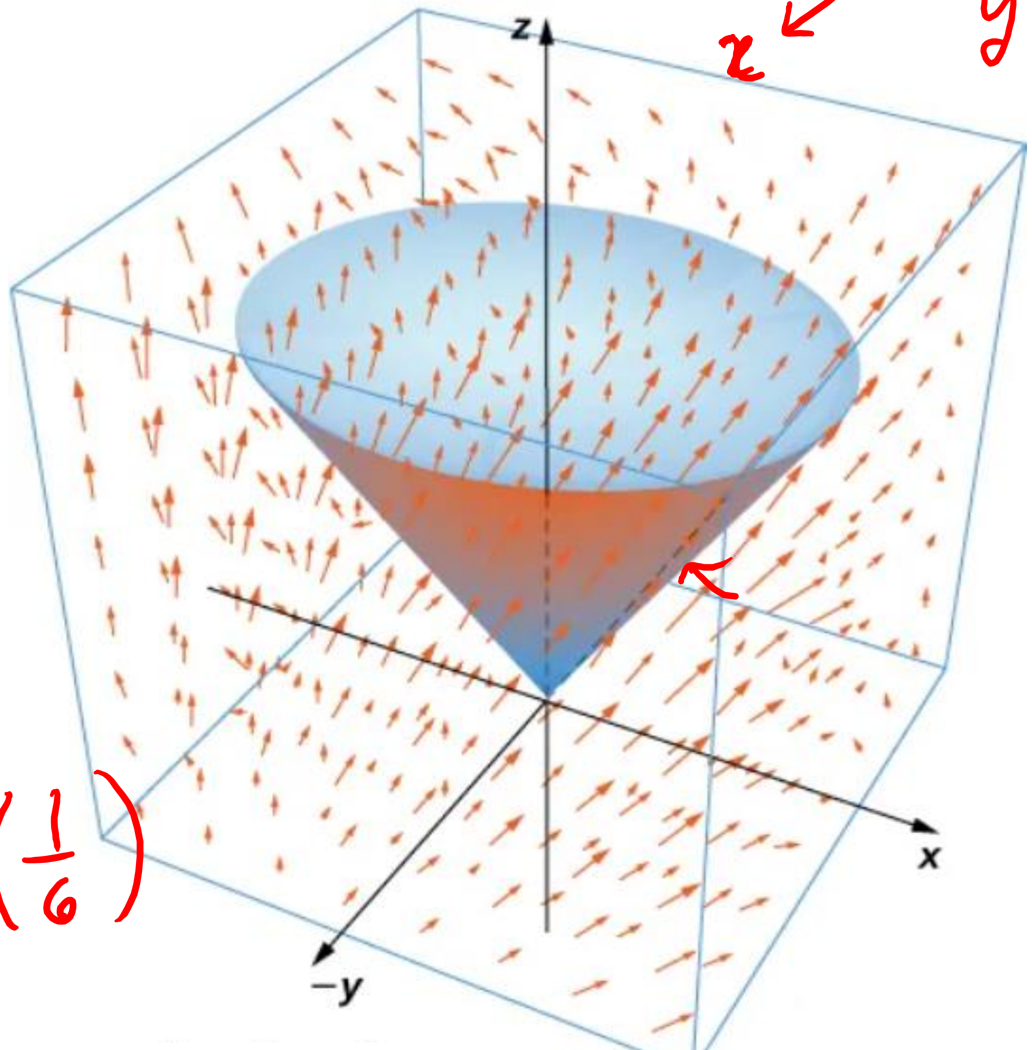
$$= 2\iiint_E dV$$

$$= 2\int_0^{2\pi}\int_0^1\int_r^1 r\,dz\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\int_0^1 r(1-r)\,dr\,d\theta$$

$$= 2\int_0^{2\pi}\left[\frac{r^2}{2}-\frac{r^3}{3}\right]_0^1 d\theta = 2\cdot 2\pi\left(\frac{1}{6}\right)$$

$$= \frac{2\pi}{3}$$



POLL 2

(Spring 2023 Final Exam #20)

$\vec{\nabla}\cdot\vec{F} = 3$

Find the outward flux of the vector $\vec{\boldsymbol{F}} = \langle \sin(y), xz, 3z \rangle$ across the boundary of the space between the two spheres of radii 1 and 2, both centered at the origin.

$a)\ 84\pi$

$b)\ 24\pi$

$c)\ 28\pi$

$d)\ \frac{28\pi}{3}$

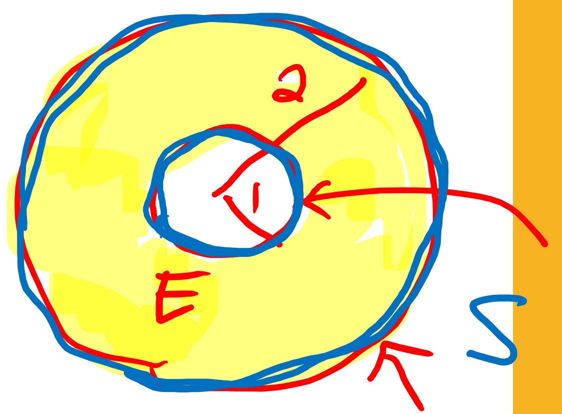
$e)\ 12\pi$

$$\iint_S \vec{F}\cdot\vec{n}\,dS = \iiint_E (\vec{\nabla}\cdot\vec{F})\,dV = 3\iiint_E dV$$

$$= 3\left[\text{volume}(S_2) - \text{volume}(S_1)\right]$$

$$= \cancel{3}\left[\frac{4\pi}{\cancel{3}}2^3 - \frac{4}{\cancel{3}}\pi 1^3\right]$$

$$= 4\pi[8-1] = 28\pi$$



POLL 3

(Fall 2019 Final Exam #18)

18. Consider $\mathbf{F} = \dfrac{\mathbf{r}}{|\mathbf{r}|^3}$, where $\mathbf{r} = \langle x, y, z\rangle$ and $|\mathbf{r}| = (x^2+y^2+z^2)^{1/2}$. Which one of the following is true

✓ (i) $\int_C \mathbf{F}\cdot d\mathbf{r}$ is independent of path. → conservative $\vec{F} = \nabla\left(\frac{1}{|\vec{r}|}\right)$

→ ~~(ii)~~ $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = 0$ for any closed surface S that encloses the origin. $= 4\pi$ for a sphere

✓ (iii) $div(\mathbf{F}) = 0$. for $(x,y,z) \neq (0,0,0)$

A. None of the above.

B. Only (i) and (ii).

C. Only (i) and (iii).

D. Only (ii) and (iii).

E. All of the above.

$$\frac{\partial}{\partial x}\left(\frac{x}{(x^2+y^2+z^2)^{3/2}}\right) = \frac{1}{r^3} + \frac{x\left(-\frac{3}{2}\right)\cdot 2x}{(x^2+y^2+z^2)^{5/2}}$$

$$= \frac{1}{r^3} - \frac{3x^2}{r^5}$$

$$\vec{\nabla}\cdot\vec{F} = \left(\frac{1}{r^3} - \frac{3x^2}{r^5}\right) + \left(\frac{1}{r^3} - \frac{3y^2}{r^5}\right) + \left(\frac{1}{r^3} - \frac{3z^2}{r^5}\right)$$

$$= \frac{3}{r^3} - \frac{3(x^2+y^2+z^2)}{r^5} = \frac{3}{r^3} - \frac{3\cancel{r^2}}{r^{\cancel{5}\,3}} = 0$$

$$= \frac{3}{r^3} - \frac{3}{r^3} = 0$$