LESSON 37

MA 26100·FALL 2023

DR. HOOD

WARM UP

(Fall 2017 Final Exam #20)

$\vec{\nabla} \cdot \vec{F} = 6xyz$

Let $\vec{F} = \langle x^2yz, xy^2z, xyz^2 \rangle$. Using the outward pointing normal on the surface S of the solid box defined by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 2$, evaluate $\iint_S \vec{F} \cdot \vec{n}\, dS$

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_E \vec{\nabla} \cdot \vec{F}\, dV$$

a) 12

b) 24

c) 9

d) 18

e) 6

$$= \int_0^1 \int_0^2 \int_0^2 6xyz\, dz\, dy\, dx$$

$$= 6 \int_0^1 \int_0^2 \left(\frac{z^2}{2}\right) xy \Big]_0^2 dy\, dx = 12 \int_0^1 x \left(\frac{y^2}{2}\right)\Big]_0^2 dx$$

$$= 24 \left[\frac{x^2}{2}\right]_0^1 = 12$$

LESSON 37

POLL 1

Evaluate $\iint_S \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{n}}\, dS$ where $\vec{\boldsymbol{F}} = \langle x^2, xy, x^3y^3 \rangle$ and S is the surface consisting of all the faces of the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes, with outward pointing normal.

$\vec{\nabla} \cdot \vec{F} = 3x$

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_E \vec{\nabla} \cdot \vec{F}\, dV$$

$$= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 3x \; dz\, dy\, dx$$

$$= \dots = \frac{1}{8}$$

a) $\frac{1}{24}$

b) $\frac{1}{12}$

c) $\frac{1}{8}$

POLL 2

(Fall 2015 Final Exam #18)

PROBLEM 18: Consider the vector field

$$\mathbf{F}(x,y,z) = \langle x^3 + xy^2 + xz^2, x^2y + y^3 + yz^2, x^2z + y^2z + z^3\rangle$$

and let S be the sphere of radius 2 centered at the origin with positive orientation, i.e. outward pointing normal. Compute the flux

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \vec{\nabla}\cdot\vec{F}\, dV$$

$$\vec{\nabla}\cdot\vec{F} = 3x^2 + y^2 + z^2 + x^2 + 3y^2 + z^2 + x^2 + y^2 + 3z^2 = 5(x^2+y^2+z^2) = 5\rho^2$$

A. -4π

B. -2π

C. $\frac{4}{3}\pi$

D. 64π

E. 128π

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{2} 5\rho^2\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\frac{\cancel{5}\rho^5}{\cancel{5}}\right]_0^2 \sin\phi\, d\phi\, d\theta$$

$$= 2^5\int_0^{2\pi}\int_0^{\pi}\sin\phi\, d\phi\, d\theta = 2^5\cdot 4\pi = 128\pi$$