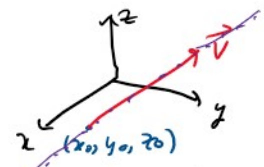
Lesson 3 - 8/25/2023 - 4:30pm

13.5: Cylinders & Quadric Surfaces – Part I

Lines & Surfaces in $\mathbb{R}^3$



We Know: Line: $\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\underbrace{\langle a, b, c\rangle}_{\text{direction vector } \vec{v}}$

in 2D: $y = b + mx$

Sphere: radius r center (x_0, y_0, z_0)

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

Plane: normal vector $\vec{n} = \langle a, b, c\rangle$

point (x_0, y_0, z_0)

$$\vec{n}\cdot(\vec{x}-\vec{x}_0) = 0$$

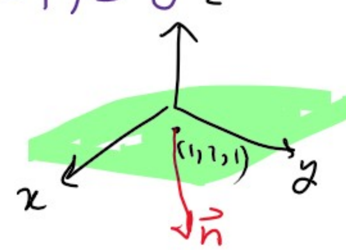
$$\langle a, b, c\rangle \cdot \langle x-x_0, y-y_0, z-z_0\rangle = 0$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

Ex: $3(x-1) + 4(y-1) - 2(z-1) = 0$

point $(1, 1, 1)$

$\vec{n} = \langle 3, 4, -2\rangle$

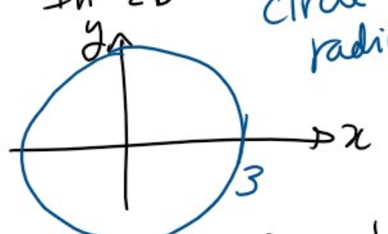


Today, more surfaces:

Ex: $x^2 + y^2 = 9$

In 2D

circle of radius 3



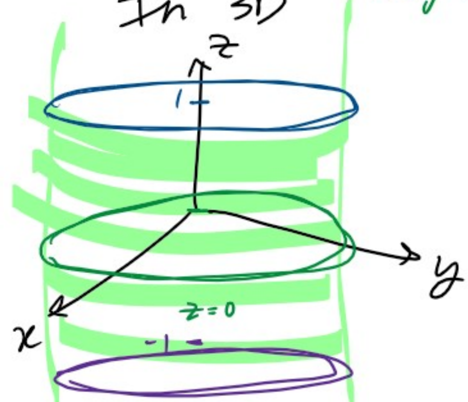
circular cylinder

infinite height

In 3D

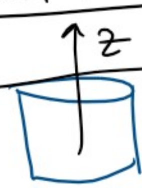
no constraint on z

z can be any #



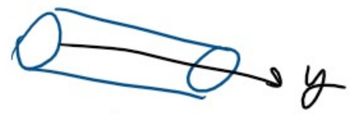
infinite height

Def: A set of curves parallel to a given axis is a <u>cylindrical surface</u> or <u>cylinder</u>



$x^2+y^2=9$

$x^2+z^2=4$



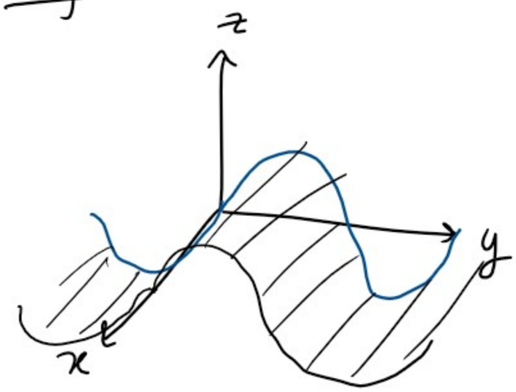
<u>NOTE:</u> <u>Any</u> curve can form a cylinder

A cylinder can have a cross-section that is <u>any</u> curve

<u>Ex:</u> $z=\sin(y)$

x is a free variable

x can be any #



$\boxed{x=0}$ $z=\sin(y)$

$\boxed{x=1}$ $z=\sin(y)$

This is a <u>cylinder</u> whose cross-section is a sinusoid

<u>Def:</u> The <u>trace</u> of a surface is the cross-section parallel to one of the coordinate planes

<u>Ex:</u> $z=\sin(y)$

<u>Trace</u>

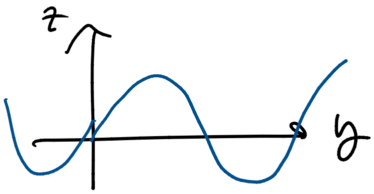
Trace $y=0$

Trace $z=0$

Trace

$\boxed{x=0}$

$z=\sin(y)$



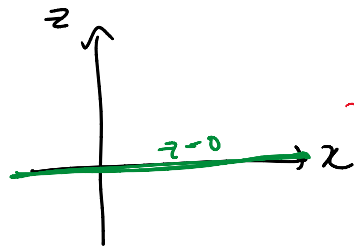
cross-section of the cylinder ☆ ☆

Trace

$y=0$

$z=\sin(0)=0$

(x any #)



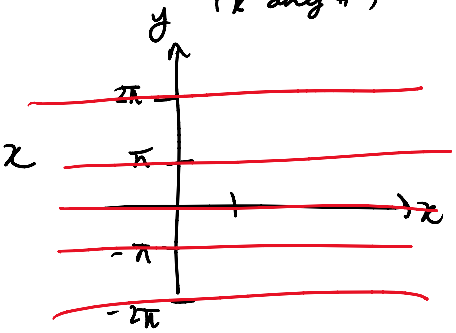
Trace

$z=0$

$0=\sin(y)$

$y=\pi, 2\pi, 3\pi, \ldots$

(x any #)



Quadric Surfaces

$$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx$$
$$+Hy+Iz+J=0$$

where $A, B, C, \ldots, J$ are constants (not all zero)

Study the 6 most common surfaces

Ex: Sketch the surface described by:

$$4x^2+9y^2+4z^2=36$$

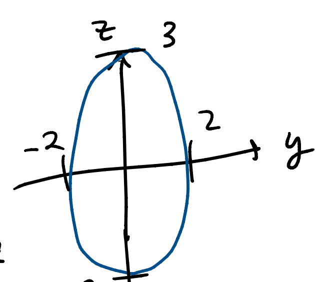
divide by 36

$$\frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{9}=1$$

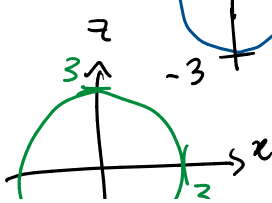
Traces:

$\boxed{x=0}$ $\frac{y^2}{4}+\frac{z^2}{9}=1$

ellipse



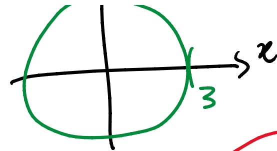
$\boxed{y=0}$ $\frac{x^2}{9}+\frac{z^2}{9}=1$



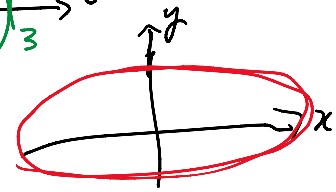
circle

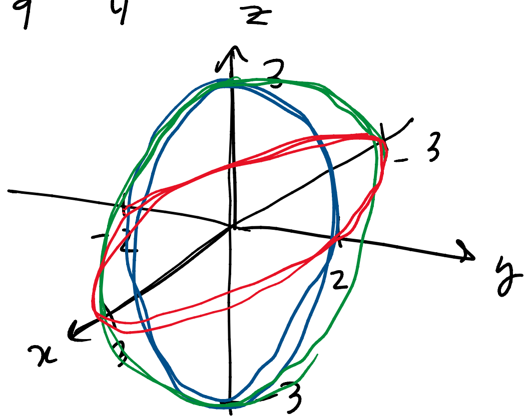
$\uparrow y$

$\boxed{y=0}$ $\quad \dfrac{x^2}{9} + \dfrac{z^2}{9} = 1$



$\boxed{z=0}$ $\quad \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$



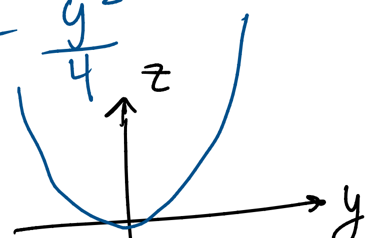


$\boxed{\text{Ellipsoid}}$

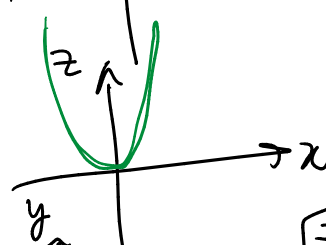
Ex: Sketch the surface

$$z = \frac{x^2}{25} + \frac{y^2}{4}$$

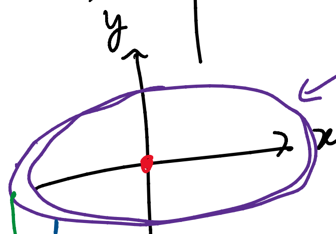
$\boxed{x=0}$ $\quad z = \dfrac{y^2}{4}$



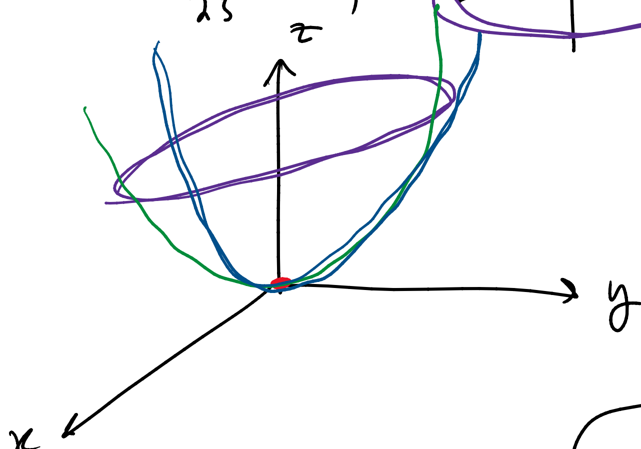
$\boxed{y=0}$ $\quad z = \dfrac{x^2}{25}$

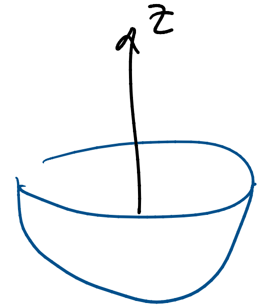


$\boxed{z=1}$ $\quad 1 = \dfrac{x^2}{25} + \dfrac{y^2}{4}$



$\boxed{z=0}$ $\quad 0 = \dfrac{x^2}{25} + \dfrac{y^2}{4}$





cup pointing up in the z-direction

$\boxed{\text{Elliptic Paraboloid}}$

x "

Elliptic Paraboloid

Quadric Surfaces - 4:30

quadric_su...

Quadratic Surfaces Notes

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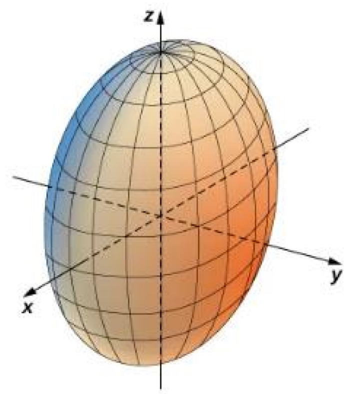
Name: Ellipsoid

Equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Traces: all are ellipses

Notes: If $a=b=c \rightarrow$ sphere

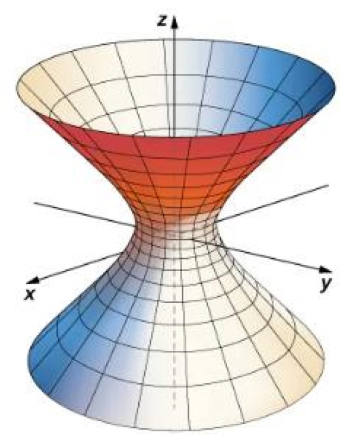


Name:

Equation:

Traces:

Notes:

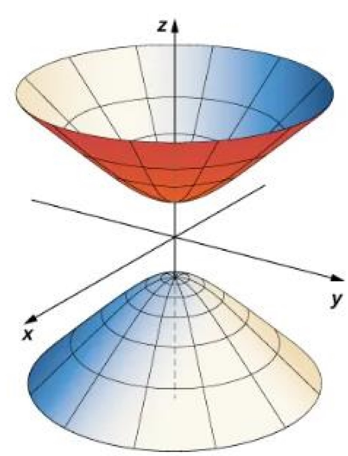


Name:

Equation:

Traces:

Notes:

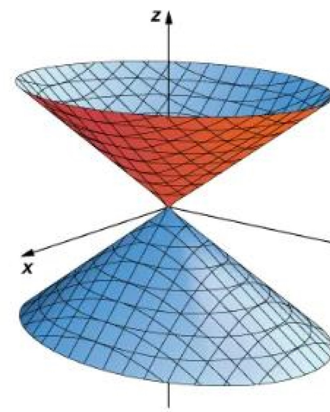


Name:

Equation:

Traces:

Notes:



Name: Elliptic Paraboloid

Equation: $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Traces:

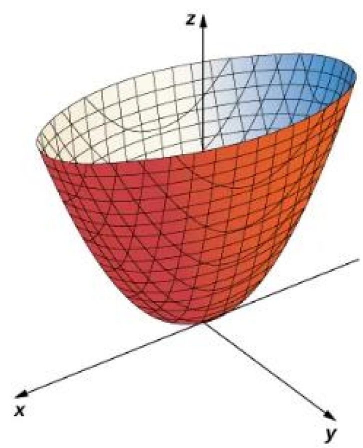
$z=\#$ ellipse

$x=0$ parabola

$y=0$ parabola

Notes:

cup is pointing toward the $+z$ axis

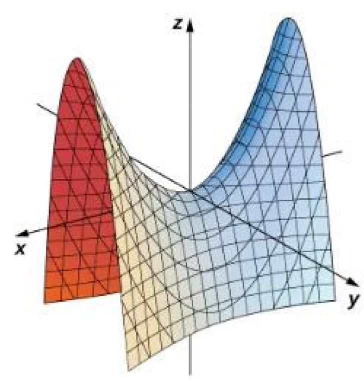


Name:

Equation:

Traces:

Notes:



*Plots of Quadric Surfaces from: Herman, Edwin and Gilbert Strang. “Calculus Volume 3.” (2018).