13.6 Quadric Surfaces - Part II

Ex: Sketch the surface

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{49} = 1$$

negative

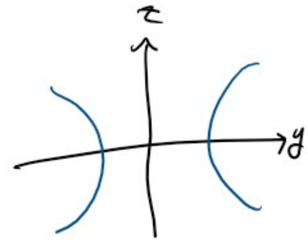
Traces: $\boxed{x = p}$

$p = \#$

$$\frac{p^2}{4} + \frac{y^2}{9} - \frac{z^2}{49} = 1$$

$$\frac{y^2}{9} - \frac{z^2}{49} = 1 - \frac{p^2}{4}$$

$\boxed{x=0} \rightarrow p=0 \rightarrow \frac{y^2}{9} - \frac{z^2}{49} = 1$

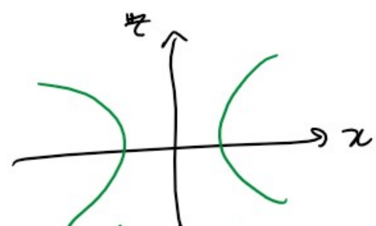


hyperbola

$\boxed{y = q}$

$$\frac{x^2}{4} + \frac{q^2}{9} - \frac{z^2}{49} = 1$$

$$\frac{x^2}{4} - \frac{z^2}{49} = 1 - \frac{q^2}{9}$$

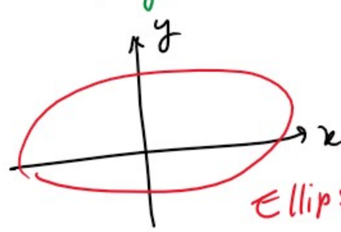


hyperbola

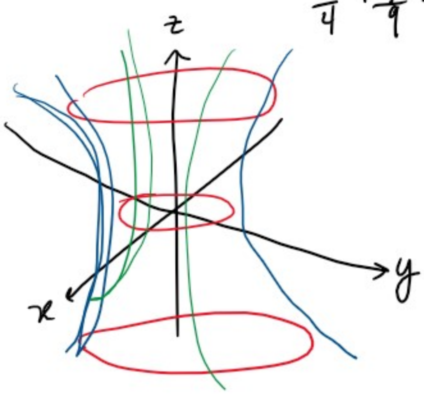
$\boxed{z = r}$

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{r^2}{49} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 + \frac{r^2}{49}$$



Ellipse



Hyperboloid of one sheet

"one sheet" - connected

Example: What happens if we make one more coeffient negative?

$$\frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{49} = 1$$

2nd neg.

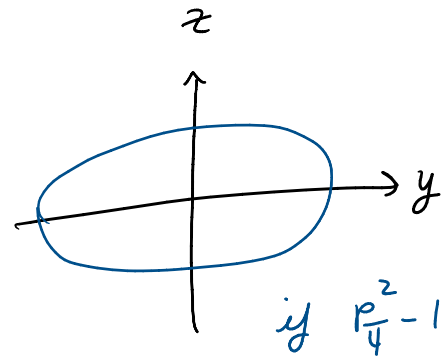
Traces:

2nd neg.

Traces:

$\boxed{x = p}$ $\quad \frac{p^2}{4} - \frac{y^2}{9} - \frac{z^2}{49} = 1$

$$\underbrace{\frac{p^2}{4} - 1}_{\text{if } \geq 0} = \frac{y^2}{9} + \frac{z^2}{49}$$



if $\frac{p^2}{4} - 1 \geq 0$

$\boxed{x = 0 = p}$ $\quad -1 = \frac{y^2}{9} + \frac{z^2}{49}$ $\Rightarrow\!\Leftarrow$ No solution

trace is empty set

gap in surface

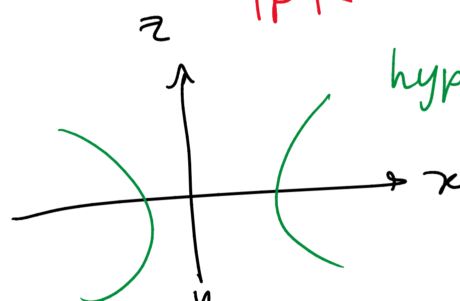
No solution when

$\frac{p^2}{4} - 1 < 0$

$|p| < 2$

$\boxed{y = q}$ $\quad \frac{x^2}{4} - \frac{q^2}{9} - \frac{z^2}{49} = 1$

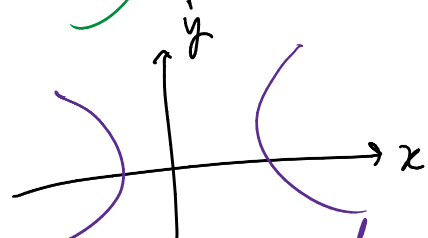
$$\frac{x^2}{4} - \frac{z^2}{49} = 1 + \frac{q^2}{9}$$



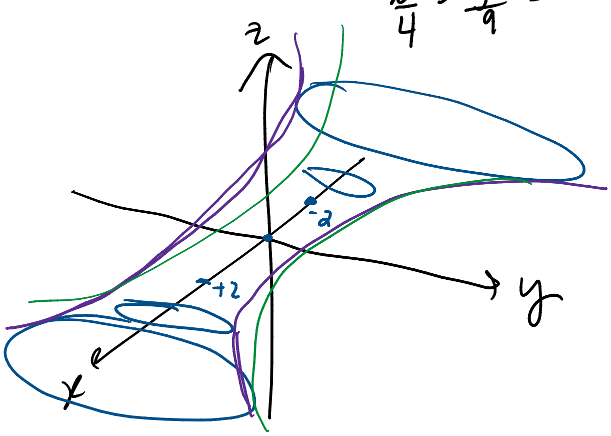
hyperbola

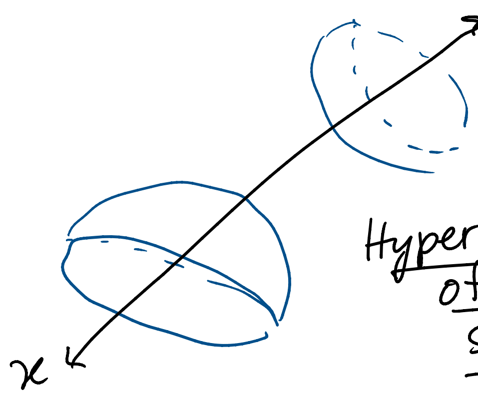
$\boxed{z = r}$ $\quad \frac{x^2}{4} - \frac{y^2}{9} - \frac{r^2}{49} = 1$

$$\frac{x^2}{4} - \frac{y^2}{9} = 1 + \frac{r^2}{49}$$



hyperbola





Hyperboloid of two sheets

"two sheets" - disconnected

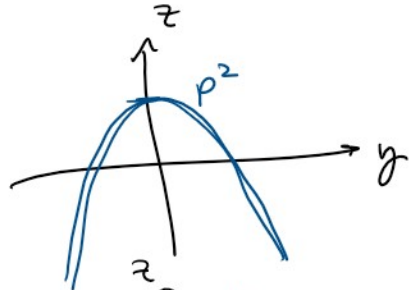
Example: Sketch the surface

Example: Sketch the surface

$$z = x^2 - 4y^2$$

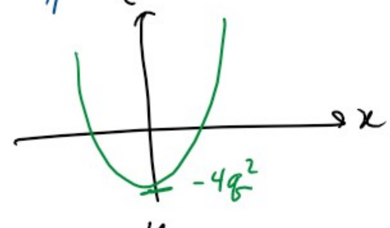
Traces: $\boxed{x=p}$ $\quad z = p^2 - 4y^2$

parabola



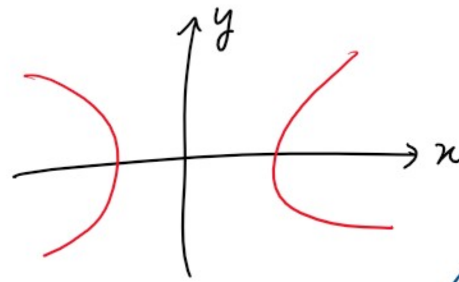
$\boxed{y=q}$ $\quad z = x^2 - 4q^2$

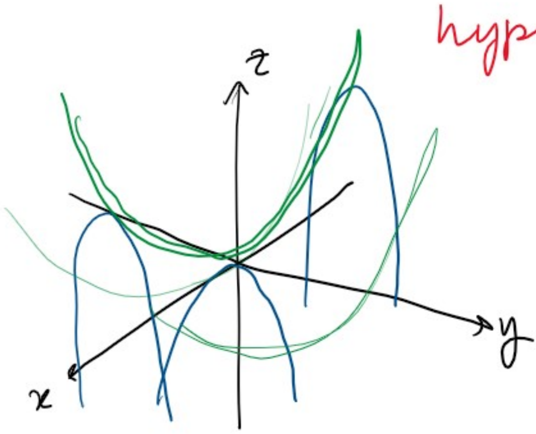
parabola



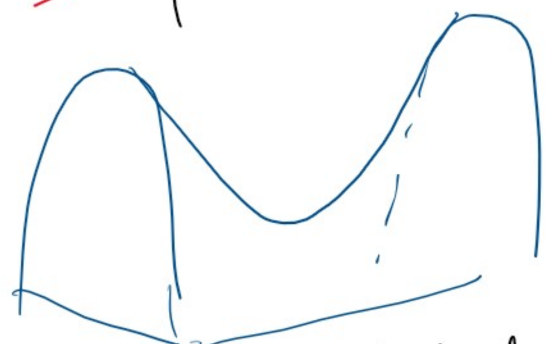
$\boxed{z=r}$ $\quad r = x^2 - 4y^2$

hyperbola





saddle



Hyperbolic paraboloid

Example: Sketch the surface

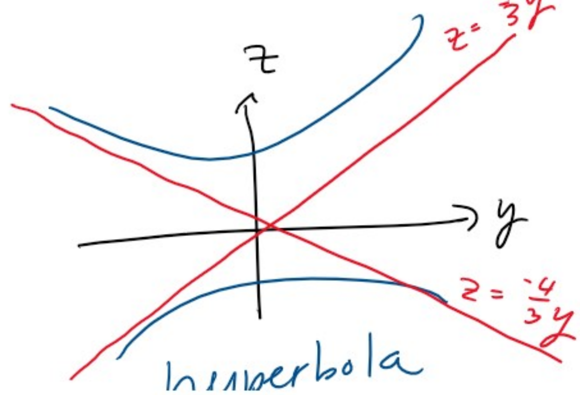
$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 0$$

← not a hyperboloid

Traces:

$\boxed{x=p}$ $\quad \frac{p^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 0$

$p > 0$ $\quad \frac{p^2}{4} = \frac{-y^2}{9} + \frac{z^2}{16}$



$p>0$ $\qquad \frac{p^2}{4} = \frac{y^2}{9} \quad 16$

hyperbola

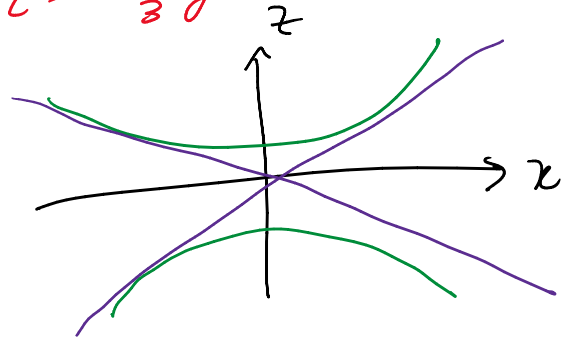
$p=0$ $\qquad 0 = -\frac{y^2}{9} + \frac{z^2}{16}$

$$z^2 = \frac{16}{9}y^2 \rightarrow z = \pm\frac{4}{3}y$$

$\boxed{y=q}$ $\qquad \frac{x^2}{4} + \frac{q^2}{9} - \frac{z^2}{16} = 0$

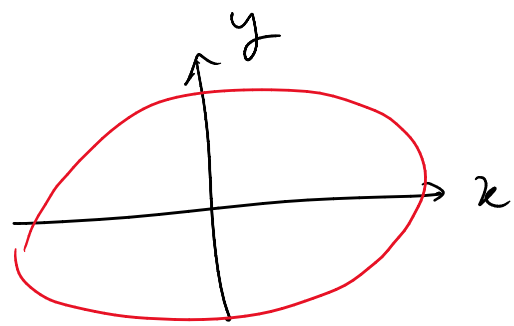
hyperbola $q>0$

$q=0$ $\quad$ X

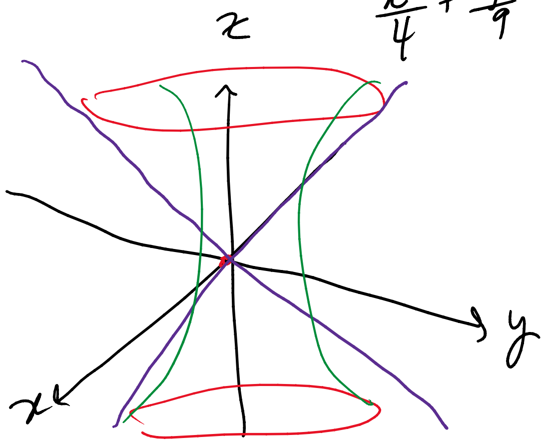


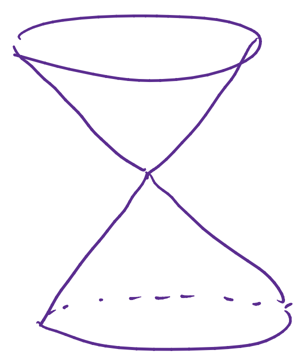
$\boxed{z=r}$ $\qquad \frac{x^2}{4} + \frac{y^2}{9} - \frac{r^2}{16} = 0$

$$\frac{x^2}{4} + \frac{y^2}{9} = \frac{r^2}{16}$$



Ellipses





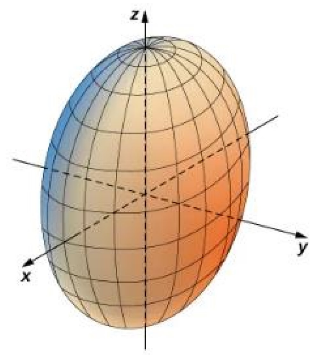
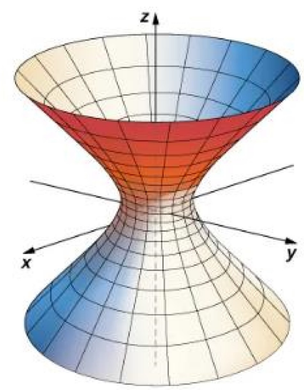
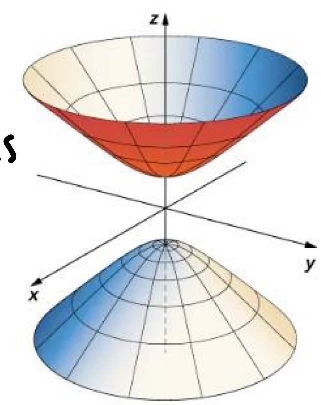
Elliptic Cone

Quadric Surfaces - 4:30

quadric_su...

Quadratic Surfaces Notes

Lessons 3 & 4 · MA 26100 · Fall 2023

Name: Ellipsoid **Equation:** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **Traces:** all are ellipses **Notes:** If $a=b=c \rightarrow$ sphere	
Name: Hyperboloid of one sheet **Equation:** $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ **Traces:** $z=p$ ellipses; $\left.\begin{matrix}x\\y\end{matrix}\right\rangle$ hyperbolas **Notes:** one neg coeff ← point in direction two pos coeff	
Name: Hyperboloid of two sheets **Equation:** $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ **Traces:** $z=p$ empty or ellipse; $\left.\begin{matrix}x\\y\end{matrix}\right\}$ hyperbolas **Notes:** two neg coeff one pos coeff – direction it points	

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Name: Elliptic Cone

Equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$

Traces:
z - ellipses
x, y } hyperbolas or X

Notes:
point in z-axis (neg coeff)

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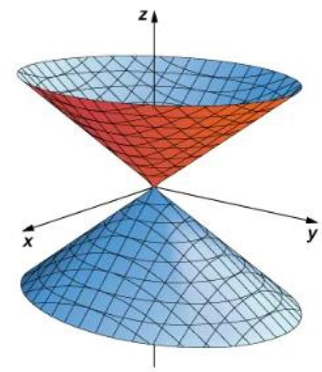

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Name: Elliptic Paraboloid

Equation: $z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

Traces:
z=# ellipse
x=0 parabola
y=0 parabola

Notes:
cup is pointing toward the +z axis

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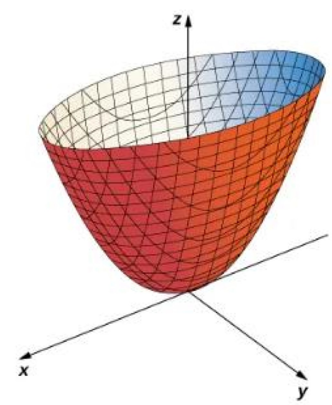

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Name: Hyperbolic paraboloid

Equation: $z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$

Traces:
z – hyperbolas
x, y } parabolas

Notes:
"parallel" to z axis

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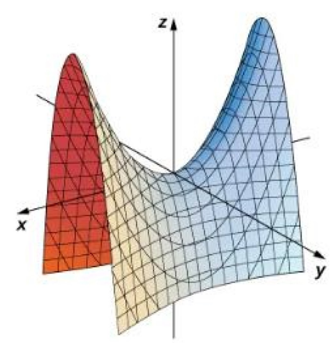

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$y=\frac{x^2}{a^2}-\frac{z^2}{c^2}$

*Plots of Quadric Surfaces from: Herman, Edwin and Gilbert Strang. "Calculus Volume 3." (2018).