LESSON 7

MA 26100-FALL 2023

DR. HOOD

LESSON 7 - WARM UP

(Spring 17 Exam 1 #3)

Suppose the trajectories of two particles are given by:

$$\overrightarrow{r_1}(t) = \langle t+1, 2\sqrt{t}, \sqrt{2}t \rangle$$

$$\overrightarrow{r_2}(t) = \langle 2t, t^2+1, t^2-2t+\sqrt{2}+1 \rangle$$

Find the angle between their tangent vectors at their point of collision.

a) 0

b) $\frac{\pi}{6}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{2}$

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

Collision:

$\vec{r}_1(t) = \vec{r}_2(t)$

$t+1 = 2t$ $\quad t=1$

$2\sqrt{t} = t^2+1$

$\sqrt{2}t = t^2-2t+\sqrt{2}+1$

$\vec{r}_1'(t=1) = \langle 1, 2\cdot\frac{1}{2}t^{-1/2}, \sqrt{2} \rangle\Big|_{t=1} = \langle 1, 1, \sqrt{2} \rangle$

$\vec{r}_2'(t=1) = \langle 2, 2t, 2t-2 \rangle\Big|_{t=1} = \langle 2, 2, 0 \rangle$

$\cos\theta = \frac{\vec{r}_1'\cdot\vec{r}_2'}{|\vec{r}_1'||\vec{r}_2'|} = \frac{\langle 1,1,\sqrt{2} \rangle \cdot \langle 2,2,0 \rangle}{\sqrt{1^2+1^2+\sqrt{2}^2}\cdot\sqrt{2^2+2^2+0^2}} = \frac{4}{2\cdot 2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

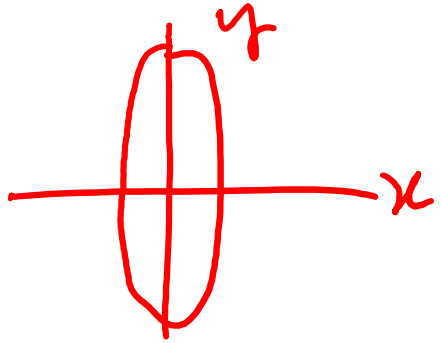
$\theta = \frac{\pi}{4}$

OFFICE HOURS

- **Dr. Hood's Office Hours:**
  - Mon, Wed, Fri at 2:00-3:00pm in MATH 844

- **TA's have office hours in the Math Resource Room (MRR)**
  - Room WTHR 182
  - Mon – Thurs 9:30am – 5:20pm and Friday 9:30am – 1:20pm
  - Schedule is posted online:
    - https://www.math.purdue.edu/academic/courses/helproom

POLL 1



elliptical motion

Determine whether the following trajectory lies on a circle:

$$\vec{\boldsymbol{r}}(t) = \langle \cos(t), 5\sin(t) \rangle$$

a) Yes, circular motion

b) No, not circular motion

$$|\vec{r}(t)| = \sqrt{(\cos t)^2 + (5\sin t)^2}$$

$$= \sqrt{\cos^2 t + 25\sin^2(t)}$$

$$= \sqrt{\cos^2(t) + \sin^2(t) + 24\sin^2(t)}$$

$$= \sqrt{1 + 24\sin^2(t)} \neq \text{const}$$

POLL 2

time of flight

The position of a cannonball is given by:

$$\vec{r}(t) = \left\langle 320\sqrt{3}t, -16t^2 + 320t + 336 \right\rangle = \langle x(t), y(t) \rangle$$

When does the cannonball hit the ground?

$y(t) = 0$

$$\frac{-16t^2 + 320t + 336 = 0}{-16}$$

$$t^2 - 20t - 21 = 0$$

$$(t - 21)(t + 1) = 0$$

$t = -1, 21$

$t = 21$

a) $t = 1s$

b) $t = 21s$

c) $t = 20s$

d) $t = 12s$

MUDDIEST POINT

What was the muddiest point from today's lecture?

a) Point of Collision
b) Circular Motion
c) Projectile Motion
d) Time of Flight
e) None – understood everything today