

Q: How do you determine if two lines share a common point?

Example: Line 1 $\langle x, y, z \rangle = \langle 0, 0, 0 \rangle + t \langle 1, 0, 0 \rangle$
 Line 2 $\langle x, y, z \rangle = \langle 0, 0, 3 \rangle + s \langle 0, 1, 0 \rangle$

note: different parameter s

set the equations equal

$$\langle 0, 0, 0 \rangle + t \langle 1, 0, 0 \rangle = \langle 0, 0, 3 \rangle + s \langle 0, 1, 0 \rangle$$

simplify each side:

$$\langle t, 0, 0 \rangle = \langle 0, s, 3 \rangle$$

write out a system of equations

$$t = 0$$

$$0 = s$$

$$0 = 3$$

\Rightarrow contradiction
 No choice of s or t will make this true

so there is no common point.

Example:

Line 1: $\langle x, y, z \rangle = \langle 0, 0, 0 \rangle + t \langle 0, 0, 1 \rangle$

Line 2: $\langle x, y, z \rangle = \langle 0, 0, 3 \rangle + s \langle 0, 1, 0 \rangle$

set equations equal

$$\langle 0, 0, 0 \rangle + t \langle 0, 0, 1 \rangle = \langle 0, 0, 3 \rangle + s \langle 0, 1, 0 \rangle$$

simplify each side:

$$\langle 0, 0, t \rangle = \langle 0, s, 3 \rangle$$

write out a system of equations

$$0 = 0$$

$$0 = s$$

this has a solution when $s = 0$ and $t = 3$

$$\left. \begin{array}{l} 0 = s \\ t = 3 \end{array} \right\} \begin{array}{l} \text{when } s=0 \\ \text{and } t=3 \end{array}$$

plug $t=3$ into Line 1:

$$\begin{aligned} \langle x, y, z \rangle &= \langle 0, 0, 0 \rangle + 3 \langle 0, 0, 1 \rangle \\ &= \langle 0, 0, 3 \rangle \end{aligned}$$

So the common point is $(0, 0, 3)$