Review Exam 1 - 10/02/2023 - 3:30pm

Review

Today we will be going over the solutions to the Practice Exam Version B

#7: Level curves of $f(x,y)=\sqrt{x^2+4y^2+4}-x$ are

$$f(x,y)=c$$

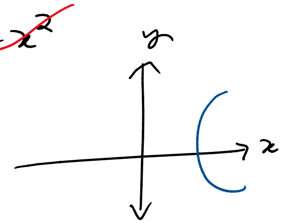
$$\sqrt{x^2+4y^2+4}-x=c$$

$$\left(\sqrt{x^2+4y^2+4}\right)^2=(c+x)^2$$

$$\cancel{x^2}+4y^2+4=c^2+2cx+\cancel{x^2}$$

$$4y^2+4-c^2=2cx$$

$$\frac{2y^2+2-\frac{c^2}{2}}{c}=x$$



#13: $f(x,y)=\frac{x^4}{4}+xy+\frac{y^4}{4}$ on $\mathbb{R}^2$

classify the c.p.

Find c.p: $f_x=0=x^3+y$

$f_y=0=x+y^3 \longrightarrow x=-y^3$

$$(-y^3)^3+y=0$$

$$-y^9+y=0$$

$$y(1-y^8)=0$$

$y=0$ or $1-y^8=0$

$y=\pm 1$

If $y=0 \longrightarrow x+\cancel{y^3}=0 \longrightarrow x=0$ $\boxed{(0,0)}$

If $y=1 \longrightarrow x+(1)^3=0 \longrightarrow x=-1$ $\boxed{(-1,1)}$

If $y=-1$ $\quad x+(-1)^3=0 \longrightarrow x=+1$ $\boxed{(1,-1)}$

2nd Deriv Test

$f_{xx}=3x^2 \qquad f_{xy}=1 \qquad f_{yy}=3y^2$

@ $(0,0)$ $\quad f_{xx}=0,\ f_{xy}=1 \quad f_{yy}=0 \quad D=0\cdot 0-1^2=-1<0$ $\boxed{\text{saddle point}}$

@ $(-1,1)$ $\quad f_{xx}=3 \quad f_{xy}=1 \quad f_{yy}=3 \quad D=3^2-1^2=8>0$, $f_{xx}>0$ $\boxed{\text{min}}$

@ $(1,-1)$ $\quad f_{xx}=3 \quad f_{xy}=1 \quad f_{yy}=3 \longrightarrow \boxed{\text{min}}$ $\quad \boxed{C}$

☆ If $f_{xx} = 0$ $\quad f_{xy} =$ $\quad f_{yy} = b$

$D = 0 \cdot b - (a)^2 = -a^2$

$\boxed{\#12}$ $(3, a, 1)$ is on the tangent plane to $f(x,y) = z = e^{xy} - 4x^2y + 3y^2$ @ $(0,1,4)$

$a=0 \quad b=1$

$f(a,b) = 4$

Explicit

Find a

Tangent Plane: $z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$$f_x = y e^{xy} - 8xy \Big|_{\substack{x=0\\y=1}} = 1 - 0 = 1$$

$$f_y = x e^{xy} - 4x^2 + 6y \Big|_{\substack{x=0\\y=1}} = 0 - 0 + 6 = 6$$

$$z = 4 + 1 \cdot (x-0) + 6(y-1)$$

$$= 4 + x + 6y - 6 = x + 6y - 2 \Big|_{\substack{x=3\\y=a\\z=1}}$$

$$1 = 3 + 6a - 2$$

$$0 = 6a \longrightarrow \boxed{a=0} \qquad \boxed{B}$$

Tangent Plane:

Explicit

$z = f(x,y)$ @ $\begin{matrix} x=a \\ y=b \end{matrix}$

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

only works for explicit form (faster)

$F(x,y,z) = f(x,y) - z$

$F_z = -1$

Implicit

$F(x,y,z) = 0$ @ $\begin{matrix} x=a \\ y=b \\ z=c \end{matrix}$

$$\nabla F(a,b,c) \cdot \langle x-a, y-b, z-c \rangle = 0$$

$$☆ \ F_x^{(a,b,c)}(x-a) + F_y^{(a,b,c)}(y-b) + F_z^{(a,b,c)}(z-c) = 0$$

$-1(z-c)$

this eqn works for explicit as well

plane is horizontal

form... (faster) z explicit as well

Tangent plane is horizontal if $\nabla F = \langle 0, 0, F_z \rangle$

$f_x = f_y = 0$

#6) $\vec{r}(t) = \langle t, \frac{t^2}{2}, \frac{t^3}{3} \rangle$ Find $K(1)$

$K = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$ $\qquad \vec{v} = \langle 1, t, t^2 \rangle \big|_{t=1} = \langle 1,1,1 \rangle$ $\quad |\vec{v}| = \sqrt{3}$

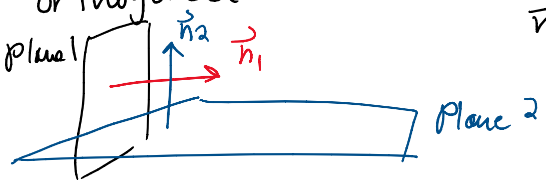
$\vec{a} = \langle 0, 1, 2t \rangle \big|_{t=1} = \langle 0,1,2 \rangle$

$$|\vec{v} \times \vec{a}| = \left| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{matrix} \right| = |\langle 1, -2, 1 \rangle| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$$

$$K = \frac{\sqrt{6}}{(\sqrt{3})^3} = \frac{\sqrt{6}}{3\sqrt{3}} = \frac{\sqrt{2}}{3}$$ $\boxed{E}$

#2) Which of the following planes are orthogonal?

$\vec{n}_1 \cdot \vec{n}_2 = 0$



A.: $x = 5z + 3y$ $\quad \vec{n}_1 = \langle -1, 3, 5 \rangle$ $\quad \vec{n}_1 \cdot \vec{n}_2 = -1 \cdot 8 + 3(-6) + 5 \cdot 2$

$8x - 6y + 2z = -1$ $\quad \vec{n}_2 = \langle 8, -6, 2 \rangle$ $\quad = -8 - 18 + 10 = -16$

NOT

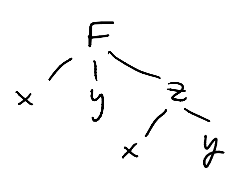
B. $x + 10y - z = 6$ $\quad \vec{n}_1 = \langle 1, 10, -1 \rangle$ $\quad \vec{n}_1 \cdot \vec{n}_2 = 1 \cdot (-9) + 10(-1) + (-1)(-19)$

$-9x - y - 19z = 2$ $\quad \vec{n}_2 = \langle -9, -1, -19 \rangle$ $\quad = -9 - 10 + 19 = 0$

#10) Suppose z is a function of x and y

$\cos(xyz) = x + 3y + 2z$

Find $\frac{\partial z}{\partial y}(0,1)$ Implicit



$F(x,y,z) = 0 = x + 3y + 2z - \cos(xyz)$

$\frac{\partial z}{} \quad \frac{-F_y}{} = \frac{-[3 + \sin(xyz) \cdot xz]}{} \Big|_{x=0}$

$F(x,y,z) = 0$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z} = \frac{-[3 + \sin(xyz)\cdot xz]}{[2 + \sin(xyz)\cdot xy]}\Bigg|_{\substack{x=0\\y=1}}$$

$$= \frac{-[3 + \sin(0)\cdot 0]}{2 + \sin(0)\cdot 0} = \boxed{-\frac{3}{2}} \quad \boxed{D}$$

If you forgot formula, use Chain Rule

$$\frac{\partial}{\partial y}\left[x + 3y + 2z - \cos(xyz)\right] = 0$$

#11 Find the maximum rate of change of $f(x,y) = \sqrt{7 - x^2 - y^2}$ at $(-2,1)$

$D_u f = \nabla f \cdot \vec{u}$

$|\vec{u}| = 1$

max rate of change $\rightarrow$ scalar $\rightarrow$ $D_u f$

where $\vec{u}$ – direction of steepest ascent

$$\vec{u} = \frac{\vec{\nabla} f}{|\nabla f|}$$

$$D_u f = \vec{\nabla} f \cdot \vec{u} = \vec{\nabla} f \cdot \left(\frac{\vec{\nabla} f}{|\nabla f|}\right) = \frac{|\nabla f|^{\cancel{2}}}{\cancel{|\nabla f|}} = |\nabla f|$$

$$\nabla f = \left\langle \frac{1}{2}(7 - x^2 - y^2)^{-1/2}\cdot(-2x),\ \frac{1}{2}(7 - x^2 - y^2)^{-1/2}\cdot(-2y) \right\rangle \Bigg|_{\substack{x=-2\\y=1}}$$

$$= \left\langle \frac{1}{\cancel{2}}(7 - 2^2 - 1^2)^{-1/2}(-\cancel{2}\cdot(-2)),\ \frac{1}{\cancel{2}}(7 - 2^2 - 1^2)^{-1/2}(-\cancel{2}\cdot 1) \right\rangle$$

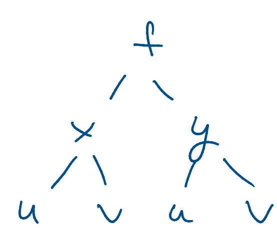
$$= \left\langle \frac{2}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right\rangle = \left\langle \sqrt{2}, -\frac{\sqrt{2}}{2} \right\rangle$$

$$|\vec{\nabla} f| = \sqrt{(\sqrt{2})^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = \sqrt{2 + \frac{2}{4}} = \sqrt{\frac{5}{2}}\ \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{10}}{2} \quad \boxed{B}$$

$$x = (u + 2v)^3 + 1$$

$$y = e^{uv} - 1$$



$$\frac{\partial f}{\partial v} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial v}$$